JMC
Joint Mathematical Council of the UK

THE TEACHING AND LEARNING OF ALGEBRA PRE-19

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Report for the Joint Mathematical Council of the UK

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The cover shows the Penrose tiling paving located outside the Andrew Wiles Building, Oxford. It is used under the Creative Commons license at File:Penrose Paving, Oxford 2.jpg - Wikimedia Commons .

Contents

Executive Summary

This report reviews current evidence on the teaching and learning of algebra up to age 19 in the UK, updating the 1997 Royal Society/Joint Mathematical Council *Teaching and Learning Algebra* report (1) in light of new research, curriculum developments, assessment practices, and the rapid expansion of digital technologies. While most of the conclusions of the 1997 report remain valid, the concerns raised in the 1997 report have been only partially addressed, and the wider context in which algebra is used has changed.

Key conclusions

Teaching and learning algebra is complex, and takes time: A secure, robust understanding of algebra develops slowly, and requires learners to engage with a wide range of related experiences woven through the fabric of the wider mathematics curriculum and over time. Continuity and coherence across education stages and school years is important. The demands this makes on teachers at all stages should not be underestimated.

Algebraic sense-making is central: Evidence strongly suggests that many learners struggle to make sense of formal algebra, often acquiring procedures without understanding relationships or meaning. This contributes to low confidence, disengagement, and limited ability or inclination to use algebra.

Early foundations are often under-used: Children bring their own rich pre-school and early years experiences into the classroom, including informal relational, spatial and pattern-based reasoning. Algebra teaching should value these 'algebraic seeds' and seek to build on them.

Progression into formal algebra needs re-balancing: Premature emphasis on formal symbolic manipulation, before secure relational foundations are established, may create persistent difficulties. Informal algebraic reasoning, language, and representation should be strengthened and extended, moving over time towards conventional notation in conjunction with robust reasoning and sense-making.

Digital technologies change, but do not remove, algebraic demands: Digital and dynamic tools can enhance conceptual understanding and modelling when well-used, but this depends on appropriate pedagogical framing. Outsourcing procedural routines increases, rather than lessens, learners' needs for deeper conceptual understanding.

System coherence matters: Algebraic curriculum content, pedagogy (including teacher education), and assessment must share common aims and values. Changes to one element of the system in isolation will not deliver sustainable improvement.

Report Recommendations

Teaching and learning. Teachers should:

- Build deliberately on early 'algebraic seeds' and pre-algebraic learning around number, measure, spatial reasoning, structure and relationships.

- Ensure continuity and coherence, especially across educational transitions, making connections to key algebraic ideas throughout the mathematics curriculum.
- Support relational understanding, generalisation, and symbol sense alongside procedural fluency.
- Attend to learners' metacognition, confidence, and enjoyment in algebra.
- *Digital futures:* make informed decisions about where and how digital technologies enhance, complement, or replace mechanical/procedural work in algebra teaching and learning.

Curriculum and assessment designers should:

- Rebalance meaning-making and procedural knowledge within algebra curricula, so that they fully support the recommended aspirations for teaching. This may require careful consideration of sequencing and curriculum coverage.
- Ensure assessment, both tasks and mark schemes, fully reflects and values this balance.
- Monitor digital assessment of algebra carefully for validity and unintended consequences, including where limitations of digital assessment may serve to narrow the taught curriculum.

Teacher education and system capacity: policymakers should

- Invest in initial and continuing teacher professional development to support rich and digitally-informed algebra teaching.
- Address other curriculum systemic constraints, including teacher recruitment and retention.
- *Digital futures:* ensure all teachers and learners of algebra have access to relevant and reliable digital infrastructure.

1. Introduction

This report reviews the teaching and learning of algebra within the current education and digital landscapes, building on the work done in the 1997 JMC/Royal Society report on the Teaching and Learning of Algebra pre-19. It differs from that report in that, for reasons that emerge below, it:

- is focused on algebraic foundations, thinking and development for the many, rather than focusing on formal algebra;
- addresses progression in algebraic thinking, knowledge and skills without tying that to specific institutional and assessment structures;
- is aimed at schools, and wider mathematics education stakeholders;
- evidences the desirability of moving to a more expansive vision for algebra education so as to securely underpin confident fluency in algebra with robust sense-making, at scale.

In Section 6 we review the implications of that approach for the conclusions reached in 1997. The review approach taken is outlined in Appendix 1, and Appendix 2 captures some key ideas and experiences for different stages in algebra learning.

We focus on the teaching and learning of algebra pre-19, particularly in the UK. Such algebra commonly includes generalisation, problem-solving and work with functions (2,3). Research characterises *algebraic thinking* as a shift from working with specific numbers and contexts towards attending to underlying structures and systems (4). **School/college algebra can therefore be seen as a set of conceptual and procedural tools for expressing generality and relationships between numbers, expressions and functions.** These relationships may be represented using symbols, graphs, tables, numbers and words, and **developing fluency in moving between these different representations is a central aim of algebra learning** (5,6).

Kieran (6) identifies **three interrelated dimensions of school algebra: *analytic* (manipulating symbols and solving equations), *structural* (recognising underlying mathematical structures, such as properties of operations), and *functional* (working with relationships between quantities using words, symbols, tables and graphs).** In practice, these dimensions are closely connected, with ***generalising* central to all three**. Learners express regularities, identify invariants, and articulate rules, often supported by different modes of representation (6).

Chazan (7) argues that curricular coherence can be strengthened by attending to the fundamental **objects of algebra** (such as variables, equations and functions) and the processes applied to them. Algebra learning, then, involves more than procedural fluency: **learners need support in understanding the *meaning* of algebraic objects and the relationships they express**, since learners may successfully operationalise procedures while remaining disconnected from algebraic meaning.

> We conceptualise these different ways of thinking about pre-university algebra as complementary lenses, each necessary to an appreciation of the complexity of the field especially as learners mature, and readers will want to draw on each at different times.

Rationale for learning algebra:

A principal use of algebra beyond school is in modelling situations, both within and beyond mathematics, in order to explain, predict and make informed decisions (2). From this perspective, algebra enables learners to represent general relationships and to reason about systems rather than isolated cases, across the mathematics curriculum and beyond. Citizens who lack confidence in expressing and manipulating generality may therefore be disadvantaged in economic, social and political contexts that increasingly rely on quantitative reasoning (2,8).

At the same time, fluency in symbolic manipulation remains central to advanced study in mathematics, as well as in cognate fields such as engineering and computing and many less intense 'user' fields. Research suggests that many middle-school learners are willing to engage with symbolic systems that do not have an immediate real-world application, provided those symbols have internal mathematical meaning for them (3). Algebra education therefore serves both instrumental purposes — supporting modelling, interpretation and use of digital tools — and disciplinary purposes within and beyond mathematics.

As argued later in this report (Section 3), contemporary learners need to both generalise and symbolise, and also to understand the power and limitations of digital technologies to which they increasingly outsource algebraic procedures (3). Developing algebraic understanding therefore involves more than technical skill: learners need to appreciate what algebra can express, when it is useful, and how symbolic representations relate to underlying relationships.

Early building blocks

Research consistently points to the importance of *relational thinking* in the development of algebraic understanding (9). However, there is substantial evidence that, historically, relational ways of thinking about mathematics have often been marginalised in favour of algebraic procedures. This has implications for learners' later ability to make sense of algebraic expressions, equations and structures.

Davydov's work (10) conceptualised algebra as building on embodied and relational experiences, such as comparing measures using e.g. 'longer than' or 'higher than'. In this approach, number and arithmetic develop from an underlying focus on relationships, rather than algebra emerging later as a generalisation of number. This perspective aligns with recent research highlighting the importance of early spatial experience in supporting later mathematical thinking more broadly (8,11,12).

There is growing interest in applying Davydov's insights to modern algebra curricula, for instance through the idea of *'seeds of algebraic thinking'* (13). This view suggests that children's pre-school experiences — including relational, spatial and comparative reasoning — can be developed in parallel with (now, usually) number learning, rather than treated as preparatory or incidental. Awareness of generality is present from children's earliest encounters with number, but these early insights require sustained and supported development to become more sophisticated over time (2,3,12).

Such exploration is often experienced as enjoyable and motivating for learners (14,15). A key challenge for teachers is whether classroom activity draws on learners' existing reasoning powers, or instead does too much of the work for them. Within current curricula, early algebra often appears as *pre-algebra*, focusing on the structure of arithmetic and generalised number behaviour — for example, properties of odd and even numbers, patterns in multiples, or the extension of number to include negative numbers. This pre-algebraic thinking underpins secure arithmetic understanding and supports later algebraic reasoning (3). Sense-making linguistic algebra tools typically develop over time, moving from everyday spoken language to increasingly abbreviated forms, and eventually towards conventional abstract algebraic symbolism and notation (16). In Section 2, we consider recent research on the pre-conceptual linguistic and cognitive knowledge that children bring to this development, and how teaching can build productively on it.

Algebra teaching therefore has the dual task of helping learners synthesise earlier experiences and knowledge into emerging forms of algebraic reasoning, while also supporting the development of technical fluency.

2. Effective teaching of algebra, including foundations for that ('pre-algebra')

The early building blocks for algebra

There is now a substantial body of evidence that children arrive at formal schooling with a wide range of mathematical and related experiences that can contribute to later algebraic thinking, including foundational relational reasoning and early forms of generalisation (4,13,14,17–19). Whether this potential is realised depends in large part on teachers' awareness of these early resources and their capacity to support deliberate building on them over time.

Walkoe and Levin (13) describe such resources as *'seeds of algebraic thinking'*: small, intuitive, often embodied ideas arising from children's pre-instructional experiences — balance, pattern recognition, covariation, 'in-betweenness', 'closing in', or replacement. Importantly, such ideas are not inherently correct or incorrect; rather, they are activated in specific contexts and may be applied productively or unproductively depending on task design and pedagogical framing (13,18). Walkoe and Levin argue for leveraging these seeds, noticing 'moments of algebraic potential', in parallel with supporting a move to algebraic understanding of number and, over time, more formal algebraic reasoning in the classroom.

The wider mathematical context for ages 3-9

Children in the UK begin formal schooling at a relatively early age, so it is important that the development of these early 'seeds' of algebraic thinking continues throughout primary education. In the early years, the emphasis is necessarily on broader mathematical development, with making mathematical meaning central to all activity. This involves children exploring mathematics in a range of contexts, including storybooks, puzzles, songs, rhymes, puppet play and games (20).

It is important that learners experience both continuity and development in the language, representations and approaches used over time. Alongside this, attention needs to be paid to their *metacognition* — their ability to monitor and reflect on their own mathematical thinking (21) — and to their *affect*, including their confidence, self-efficacy and enjoyment in mathematics (5,20). These characteristics are supported through frequent discussion, purposeful questioning, and classroom environments that are secure, discursive, and both challenging and affirming, using tasks such as those suggested in Figure 1. Such classrooms can pre-empt the damaging development of mathematics anxiety (22).

Teaching should build on children's existing linguistic and cognitive resources, encouraging them to articulate what they know, reflect on their reasoning, and communicate their ideas to others. In doing so, it lays foundations for algebraic thinking through problem-solving, modelling, generalising and early functional reasoning. Throughout, supporting children to make mathematical sense of the world around them and of what they are taught is central to empowering them as learners (17).

A key aspect of this sense-making is establishing clear connections between numbers, quantities and the relationships between them. For example, children should be encouraged to see one-to-many correspondences as the basis for multiplicative relationships. Research suggests that representing relationships using numbers is more demanding for learners than representing quantities alone, and therefore requires deliberate and sustained focus. In particular, attention should be paid to building experience of the equals sign as both operational and relational, to underpin later algebraic use (23): $4 + 5 = ?$ as well as $8 + 4 = \square + 5$.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

100-squares can be used as the basis for number structure and algebraic exploration from early primary through to advanced school mathematics, adapted to learners' stage and needs. (They can also support a range of number conceptual work – as a 'race game up or down the grid, re-assembling a cut-up grid, fractions, decimals, percentages). Teachers can support learners to explore, conjecture, justify, discuss, question and expand the ideas of others, and become more conscious of their monitoring and control of related thinking ('metacognition').

-In early number work, the structure supports emerging grasp of sequence and place value, addition and subtraction. What patterns do learners see? Why does the 100 square work like that? Point to a number. Where is one more/less? two more/less? ten more/less? what about nine more/less? why?

- *Learners can find, and justify, 'short cuts' to counting on, or back, in ones. What have they done, why?*
- *They can explore patterns in multiples and factors: can they explain what they see?*
- *Cover a number with a counter. What is the missing number? How many ways can learners explain how they know?*
- *Find the missing numbers in a 'piece taken from a 100 square', e.g. as shown here. Different shapes, and gaps, make different demands. How do learners explain their reasoning?*

12		14
	23	

- *Once learners are becoming more confident with algebraic notation, explore the sums (or products) of the diagonals of a 2×2 square on a 100-grid. How can that be explained? What about the diagonals of other size rectangles? Or sums or other relationships of the numbers in T-shapes of different sizes?*

Figure 1: Thinking about structure with a 100 square: explorations from early primary through to advanced school mathematics. Source: Authors

Effective teaching encourages children to represent problems in ways that make sense to them, and to monitor, reflect on and communicate their thinking so that both learners and teachers can identify and address differing conceptions. Feedback should be specific and clear, focused on supporting further effort, and used judiciously. Learning is supported through the use of physical manipulatives, alongside a gradual introduction to more standard representations, such as number lines and early steps towards graphical representation (13,17,20).

Pre-algebraic thinking is developed when teachers repeatedly draw attention to underlying structures and patterns across a range of contexts, building on early years' experiences with pattern (24). For example,

a) Decide whether each statement is always, sometimes or never true. (How do you know?):
(i) If a number is a multiple of 10, it is also a multiple of 5.
(ii) If a number is a multiple of 4, it is also a multiple of 8.
(iii) If a number is a multiple of 9, it is also a multiple of 2.
(iv) Multiples are positive integers.

b) Is it always, sometimes or never true that adding two consecutive multiples of 5 will give a multiple of 10?

c) Is it always, sometimes or never true that adding five consecutive multiples of 2 will give a multiple of 10?

Figure 2: Pre-algebra discussion task. Source: DfE Mathematics Guidance: Key Stage 3 (2021) p35

this may involve ordering and comparing quantities, following and articulating rules, recognising that collections can be counted together in any order, and gradually supporting movement towards more abstract understanding of number and numerical and spatial relationships, including through informal measures (12,13,17,20). An example is given in Figure 2.

Developing algebraic ideas through ages 7-14

As learners develop, pattern recognition and pattern-generation tasks are often more powerful when they are culturally meaningful and situated in familiar contexts. Carefully structured teaching can then build on these experiences to support early functional thinking (12,25). This kind of sense-making should continue to underpin a gradual transition towards foundational school algebra (3,5), with teaching consistently exposing relationships as well as procedures (19). Even where the curriculum separates algebra as a distinct strand, early algebraic experiences necessarily draw on learners' prior mathematical and wider knowledge, including earlier work with number, shape and measure. At the same time, experiences that develop relational and measure-based reasoning, including use of manipulatives and a variety of other representations, should continue alongside number work, supporting a coherent progression into formal algebra (3,5).

Development of algebraic language

The abstract and symbolic aspects of algebra should develop alongside learners' understanding of their meaning and their role in expressing generality. This progression is supported by learners explicitly articulating relationships such as ordering, proximity, balance and imbalance, across spatial experiences, measure and number, as well as gaining wide experience of communication around sequencing and reversing operations (3,5,12,13). Natural language provides an effective legitimate starting point for expressing generality, and symbolic algebra should be introduced gradually and with clear purpose (3).

Carefully chosen examples can hint at underlying algebraic ideas, while providing time for learners to articulate their thinking verbally supports movement towards informal, and later formal, symbolic notation (26). Figure 3 shows a set of such examples drawn from learners' own 'clues'. When teaching prioritises relationships rather than procedures, even young learners are able to work with informal literal symbols, interpret the equals sign relationally, and express simple functional rules in words (19). Conjecturing about relationships — particularly those involving operations and their properties — and justifying such conjectures offers a powerful route into early algebraic thinking (3,27).

Learner-generated representations — such as drawings, diagrams, manipulatives and story contexts — play an important role in supporting early algebraic reasoning, particularly in relation to generalised arithmetic and reasoning about operations. Working with multiple representations strengthens learners' ability to move between concrete experience and abstract thinking, laying important foundations for later work with algebraic notation and proof (28). Secure progress depends on explicit attention to mathematical structure and on the careful design and use of representational supports. Both curriculum and pedagogy strongly influence the strategies learners adopt, especially in equation solving and functional reasoning (19). Opportunities for nurturing algebraic procedural flexibility can, and should, be found and deliberately exploited — through structured classroom dialogue, flexible use of representations, and collaborative reasoning: they can be powerful in supporting the development of algebraic thinking, as well as building confidence in procedures (19).

Parvi asked her class to each 'think of a number' and write down a clue to the number. She gave an example of her 'teacher's number' that she called T, 'for short'. Her 'clue' was that 3 times the number made 18. She modelled her 'laziness' to write down 3 × T =18, and asked the class how they could work out what number she had thought of. She then asked how she could make her clue harder, and collected suggestions, asking the class how she might write each down. Parvi then collected and deliberately ordered a variety of clues developed by learners. The class attempted and discussed solutions, with the 'owner' adjudicating responses:

W + 11 = 20; 2 × A = 28; 6 × T + 3 = 15; 4 × B - 5 = 7; 6 × E + 2 = 17.....

Figure 3: Steps towards equation- and symbol-sense (Source: Authors)

As learners develop mathematically, they require a balance of procedural and conceptual approaches, combining explicit teaching with opportunities for problem-based learning. Note the activity shown in Figure 3 gives learners 'ownership' of a task, gives the teacher insights into learners' number structure and arithmetic confidence, offers opportunities to develop meaning for '=' and build foundations for solving equations, and challenges assumptions that 'a number' has to be a positive integer – as well as offering scope for meaning-making discussions and their reflection in informal notation.

Learners may often appear confident to use symbols while remaining disconnected from their underlying meaning. Arcavi (29) characterises this challenge as the need to develop *symbol sense*: the ability to flexibly 'zoom in' on symbolic manipulation when appropriate, and 'zoom out' to consider relationships and problem contexts.

Developing symbol sense depends on integrated understanding of quantities, relationships and symbols, with particular attention to additive and multiplicative structures, inverse processes, and clear communication about the order of operations. Teachers need to build explicit connections between arithmetic reasoning and algebraic symbolism, rather than assuming that learners will naturally generalise from inverse operations or calculation procedures to symbolic manipulation. Sustained and varied engagement with variables and representations of unknowns can help prevent common misinterpretations and support the development of conceptual understanding (30) – see Figure 4.

For example, letters may take on different roles in algebra — as unknowns, variables, constants or parameters — and learners also need opportunities to work with both ideas of equality and equivalence. Algebra therefore involves more than generalised arithmetic, and learners benefit from extensive and deliberately varied experiences that support them in making sense of these different roles and meanings (4). Sustained and varied engagement with variables and representations of unknowns can help prevent common misinterpretations and support the development of conceptual understanding (30). Difficulties can arise when algebra is experienced mainly as a set of rules to follow. Learners need time both to make sense of the underlying operations and to become fluent with the associated notation, and this development cannot be rushed (2).

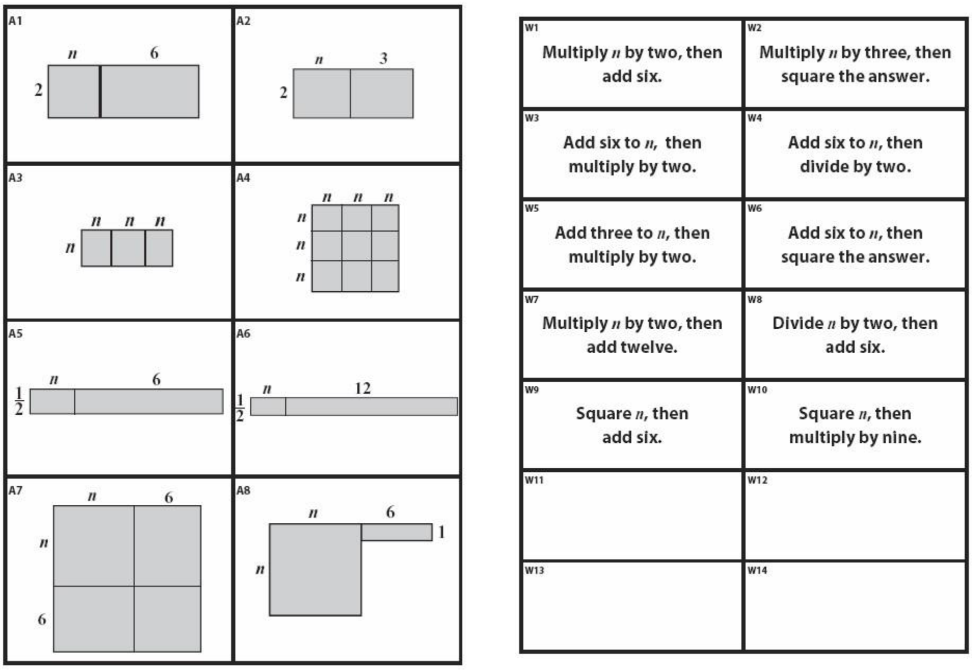

Figure 4: Matching cards of different algebraic representations (tables of values and algebraic expressions also provided, with gaps for learners to provide missing representations)

Source: DfEE Improving Learning in Mathematics https://www.stem.org.uk/resources/library/resource/26952/interpreting-algebraic-expressions-a1

Learners develop understanding of algebraic structure through multimodal, embodied activity, including gestures, manipulating objects, sliding, grouping and separating. The transition from these embodied experiences to more conceptual understanding can be supported by explicitly naming operations (such as ‘remove’ or ‘separate’), and by using story problems and narratives to help learners visualise and reason about equivalence (31). Contexts, models and metaphors provide important scaffolding during the process of abstraction; however, teachers need to make informed decisions about when to introduce such support and when to withdraw it. Central to this is sustained attention to learners’ meaning-making as they encounter algebraic expressions and equations represented in different forms, including function machines, tables, patterns, graphs and shapes (16).

Equations

There are two commonly used approaches to working with equations, both grounded in ideas of equivalence. One treats an equation as a sequence of operations to be undone using inverse operations, while the other conceptualises equality as a balance that must be maintained. Deciding which approach is most productive for a particular equation can be challenging, but learners benefit from engaging with both over time, as each supports different aspects of later algebraic work. Learners should also experience graphical approaches to solving equations, which offer further opportunities for developing understanding and have rich potential for future learning (3).

Worked examples can support learning by enabling learners to analyse algebraic reasoning and strategies (32). Learners should be encouraged to talk about representations, attend to their structure, and read expressions relationally, becoming aware that different representations highlight different aspects of the mathematics. When solving problems, teachers can support learners to choose deliberately between alternative algebraic strategies, to justify their choices, and to compare different approaches (3,4,32) — see Figure 5. Importantly, such prompts target learners' *metacognition*, helping them reflect on what they are learning and to generalise from their experience (21).

Same problem solved using two different solution strategies*

Strategy 1: Devon's solution—apply distributive property first	
Solution steps	**Labeled steps**
$10(y+2)=6(y+2)+16$	Distribute
$10y+20=6y+12+16$	Combine like terms
$10y+20=6y+28$	Subtract $6y$ from both sides
$4y+20=28$	Subtract 20 from both sides
$4y=8$	Divide by 4 on both sides
$y=2$	

Strategy 2: Elena's solution—collect like terms first	
Solution steps	**Labeled steps**
$10(y+2)=6(y+2)+16$	Subtract $6(y+2)$ on both sides
$4(y+2)=16$	Divide by 4 on both sides
$y+2=4$	Subtract 2 from both sides
$y=2$	

Prompts to accompany the comparison of problems, strategies, and solutions

- What similarities do you notice? What differences do you notice?
- To solve this problem, what did each person do first? Is that valid mathematically? Was that useful in this problem?
- What connections do you see between the two examples?
- How was Devon reasoning through the problem? How was Elena reasoning through the problem?
- What were they doing differently? How was their reasoning similar?
- Did they both get the correct solution?
- Will Devon's strategy always work? What about Elena's? Is there another reasonable strategy?
- Which strategy do you prefer? Why?

*Adapted from Rittle-Johnson and Star (2007).

Figure 5: Prompts for supporting learning from the use of different strategies (Source: (32))

Over time, learners should use algebra purposefully across a range of situations, beginning to construct algebraic models for different contexts. As part of this, they can benefit from opportunities to explore and use algebraic manipulation software, as discussed further in Section 3.

Algebraic experiences through to pre-university

It is important that learners experience continuity in the emphasis on meaning-making and relationships as they develop more advanced algebraic thinking, including conjecturing and reasoning – broadly speaking, similar, but stage-appropriate, approaches continue to underpin algebraic progression that is underpinned by confident learner sense-making (3,4). We discuss related shifts in needs in Section 3, and the curriculum system implications in Section 5. Longer-term success in algebra is supported by attention to key themes such as mental imagery, the interplay of freedom and constraint, recognising invariance amidst change, doing and undoing, characterising and organising mathematical ideas, extending meaning, and developing a language for expressing generality (2). Such themes are part of a learner's 'structure sense' (6,33,34).

Authentic mathematical reasoning often involves modelling. This typically includes starting with a situation — real, imagined, or purely mathematical — identifying its essential features and relationships, and representing these algebraically through meaningful equations or inequalities. Learners then manipulate these expressions to address the mathematical questions that arise, for example by solving equations or inequalities, isolating variables, or identifying particular solutions. Finally, solutions are interpreted and tested against the original situation for appropriateness (2).

From this perspective, symbolic manipulation is not the central purpose of algebra teaching but a by-product of meaningful reasoning (2,3). Well-designed tasks should therefore genuinely *require* algebra, with algebraic reasoning — such as reasoning using inverse operations — arising naturally from the problem context (3).

There is less research on how higher levels of school algebra are learned, but 'what there is supports the view that understanding and anticipating the purpose and relational meaning of manipulations makes a difference to the learning for most students' (2, p26).

Working with functions

The teaching and learning of functions is an area of the algebra curriculum that has received substantial research attention, although there is limited consensus about optimal methods or curriculum sequencing (3). There is some evidence that younger learners can work productively with tables of values, function machines and similar representations when these are situated in familiar and meaningful contexts. However, there is stronger agreement that attention to the properties and behaviour of functions should precede the introduction of formal definitions (3).

Developing a secure understanding of functions — both as mathematical objects and as tools for modelling — takes many years and requires varied experience. This includes modelling situations, interpreting functional relationships, translating between representations, treating functions as objects, and acting on them using procedures. Tasks that involve matching graphs to real-world situations can create productive cognitive conflict and thus support learning (Figure 6). Over time, consistent use of multiple-representation software, combined with carefully designed pedagogical tasks, can help learners develop understanding of key functional concepts and properties (3) (see Section 3).

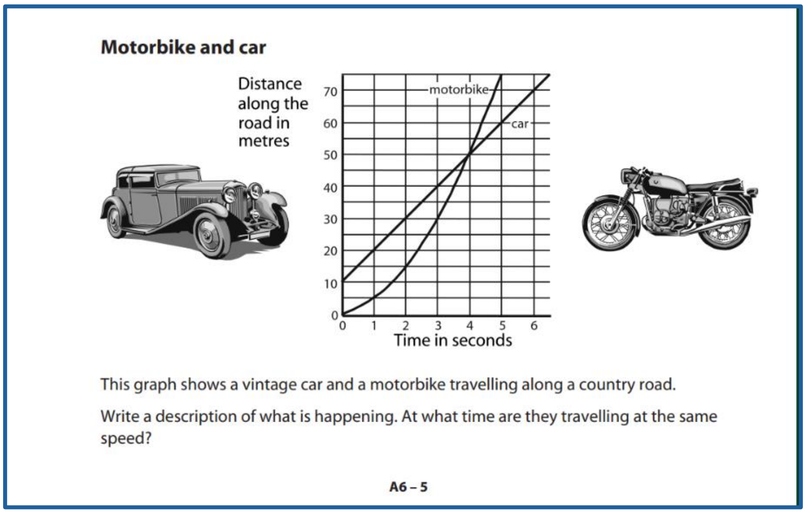

Figure 6: Interpreting real-life graphs. Source: DfEE Improving Learning in Mathematics A-6 https://www.stem.org.uk/resources/library/resource/28924/interpreting-distance-time-graphs-a6-suitable-for-home-teaching

Working with functions is a complex and long-term endeavour: at various stages in the school curriculum functions can be represented by:

- 1-1 or many-1 mappings between sets (sometimes non-mathematical sets)
- Input/output machines with algebraic workings
- Expressions to calculate y-values from x-values

- Relations between particular x- and y-values (e.g. in a table of (x, y) pairs)
- Relations between a domain of x-values and a range of function values (e.g. $f(x) = 0$ when x is rational, $f(x) = x$ when x is irrational)
- Representations of relations between variables in 'realistic' situations
- Graphs that depict particular values (discrete points marked)
- Graphs that have particular characteristics (turning points, roots etc.)
- Graphs that can be transformed by scaling, translations etc.
- Structures of variables defined by parameters and relations

i.e. there are multiple ways of seeing graphs – from calculations (x to y) through to mathematical families and objects ($y = ax^2 + bx + c$ – the object that is the family of quadratics) and importantly, moving to use of variables other than x and y (3, pp173-175).

Recommendations

Teachers should:

- Build deliberately on early 'algebraic seeds' and pre-algebraic learning around number, measure, spatial reasoning, structure and relationships.
- Ensure continuity and coherence, especially across educational transitions. Make connections to key algebraic ideas throughout the mathematics curriculum.
- Support relational understanding, generalisation, and symbol sense alongside procedural fluency.
- Attend to learners' metacognition, confidence, and enjoyment in algebra.

3. Developments in algebraic needs post-1997

Increasing digital affordances have significant implications for both the teaching and learning of algebra, and for mathematics curricula more broadly. Related *techno-mathematical literacies* are highly contextualised, and must address the complexity that can surround even relatively simple algebraic activity in workplace and everyday settings (35). At the same time, persistent inequities in access to reliable bandwidth and appropriate hardware — at both school and individual learner levels — remain a serious concern, with clear risks of widening existing inequalities (36,37).

Rapid developments in digital technologies should prompt reconsideration of curriculum priorities, classroom practices and assessment models, particularly as routine procedures become increasingly automated and greater emphasis is placed on higher-order reasoning (36). These shifts can affect not only the nature of classroom tasks, but also patterns of interaction between teachers and learners, potentially redistributing mathematical authority and widening classroom attention to global or applied mathematical perspectives (36,38).

While the mathematics itself has not changed, the ways in which mathematics is used have shifted, requiring a rebalancing between procedural fluency and quantitative reasoning. Algebra curricula therefore need to reflect the growing value placed on problem-solving, reasoning, creativity and conceptual understanding (36,37). The expanding use of artificial intelligence further raises ethical, pedagogical and assessment-related challenges (37). As a result, learners increasingly require deep mathematical understanding that can be applied flexibly across contexts — a clearly complex set of goals (35). Immediate recommendations are outlined in a recent JMC report (39), with wider systemic implications discussed in Section 5.

Use of digital technologies for the teaching and learning of algebra

Teachers can use digital technologies for a range of algebraic purposes. These include supporting conceptual understanding, outsourcing routine algorithmic processes (for example through the use of hand-held calculators), and extending learners' experience of school algebra. Digital tools are also used to support wider educational, employment and personal needs, including applications that connect algebra with fields such as art, linguistics, the humanities and the social sciences (40).

Some uses of digital technologies for algebra learning are well established in parts of the UK, including work with spreadsheets, graphing software, programming environments and databases. In effective practice, learners are supported not only to use these tools, but also to understand, adapt and control them. However, impactful use depends on careful pedagogical planning — including clear learning goals and task design — and places considerable demands on teachers' knowledge and skill (3,36). When learners engage with digital mathematical environments for algebraic exploration and discovery, they typically require active support in learning to use the technology, and tasks need to be deliberately structured to suit, and capitalise on, the digital environment (41).

Despite this potential, progress in classroom use of such technologies in much of the UK has plateaued in recent years (39). Although the uptake of technology in mathematics classrooms accelerated rapidly following the onset of the COVID-19 pandemic, this was often for organisational or presentational purposes rather than primarily targeted at mathematical purposes (37). In England, the use of calculators in primary schools to support pre-algebraic thinking is currently discouraged, whereas in Scotland such use is more common. There is scope for further development of calculator use across the UK, alongside earlier and more systematic use of graphing and spreadsheet software to support algebraic thinking (39).

There is strong evidence that the informed use of digital and dynamic tools can support learners in making sense of algebraic representations and notation, particularly when such tools are explicitly linked to underlying (pre-)algebraic structures (37,42). For example, programs and spreadsheets can help learners come to see the equals sign as expressing relationships between quantities, rather than simply signalling a command to calculate (4). Digital environments can also enable teachers to foreground real-world mathematical applications and to explore alternative forms of assessment.

The rapid and widespread adoption of digital technologies has the potential to reshape both formative and summative assessment in algebra. However, this potential is unevenly realised, as significant barriers remain in relation to access to reliable bandwidth and appropriate hardware at both institutional and individual learner levels. Where digital assessment is implemented effectively, it can offer increased flexibility in where and when assessment takes place, enable interactive task design, support automated scoring and feedback, and allow for adaptive testing approaches. Where automated scoring is used, the quality of analysis and feedback is important if assessments are to be used by learners and teachers to inform future learning.

Digital assessment can take a range of forms, from *assessment with technology* — for example, traditional paper-and-pen assessments in which learners may choose when to use a graphing calculator — to *assessment through digital technology*, where the assessment itself is mediated by digital tools (35). Each approach raises important questions about validity, equity and alignment with valued learning outcomes, particularly in relation to algebraic reasoning and the appropriate use of digital tools.

However, the automated scoring of complex mathematical responses remains challenging, particularly for assessment items that go beyond basic skills. As a result, there is a continuing risk that what is assessed digitally will be what is easiest to score — typically procedural knowledge — rather than what is most highly valued educationally, such as reasoning, interpretation and conceptual understanding. Further development is therefore needed in this demanding area of assessment design (35). Further, if appropriate and purposeful use of digital tools is genuinely valued in the learning of algebra, assessments should be designed to encourage — or in some cases require — their authentic use. Systemic implications of advances in use of digital approaches for teaching, learning and assessing algebra are discussed in Section 5.

Tools such as WolframAlpha and Desmos *can* shift classroom emphasis away from manual execution of procedures towards developing conceptual understanding and reasoning. However, there remain concerns about the risks of 'black-box thinking' and the potential loss of both procedural and conceptual fluency (37), particularly as traditional algebraic activities — such as symbolic manipulation or graphing — can increasingly be outsourced to powerful technologies, including computer algebra systems (CAS), AI models, multitouch and immersive environments, and remote learning platforms (36). Effective integration therefore depends on careful pedagogical choices about when digital tools should replace, complement, or extend learners' own mathematical work.

Graphing software

Graphing software can help focus learners' attention on important features and overarching relationships, while supporting processes of checking, trialling and refinement in (pre-) algebraic work. It can also promote learner independence and support peer discussion and exchange (43). Working with multiple representations within a computational environment supports understanding of substitution, as well as connections between symbolic expressions, functional properties and graphical representations (3).

Such environments make use of dynamic imagery, encouraging learners to attend closely to the display and interpretation of relationships, thereby supporting developing understanding and facility with functions (2). Effective use requires tasks that are clearly aligned with teaching goals and designed to make purposeful use of the available interactivity. A shared repertoire of graphing techniques can further support learning, while curriculum materials and scripts should incorporate these features to enable both proactive structuring and responsive shaping of learner activity (41).

Computer algebra systems

Computer algebra systems (CAS) integrate symbolic manipulation with graphical, numerical and tabular representations, often in interaction with spreadsheets and dynamic geometry software. Classrooms that incorporate CAS create opportunities for learners to explore mathematical invariants, link multiple dynamic representations, engage with real data, and simulate both real-world and mathematical relationships (38). However, there is internationally at present limited evidence of large-scale or systemic adoption and impact of CAS in school settings (38,39).

At the same time, there are longstanding debates about the extent to which certain manual skills remain essential, including algebraic manipulation and graphing by hand. While digital graphing tools can produce accurate graphs quickly, hand-drawn graphs are argued to support foundational intuition, strengthen conceptual understanding of functions, and contribute to the development of symbol sense (43,44). In contrast, plotting technologies tend to present graphs and tables as static objects, rather than emphasising the dynamic processes through which these representations are generated (2). Effective use of digital tools including CAS therefore requires careful consideration of how digital tools complement, rather than replace, learners' own mathematical reasoning, procedures and decision-making.

Digital tools *can* support algebraic learning in meaningful ways, with studies reporting positive effect sizes for digital-technology-supported mathematics learning typically ranging from approximately 0.38 to 0.65 (36). However, much of this evidence comes from settings where technology is used by particularly motivated teachers or 'digital enthusiasts', which limits how far findings can be readily generalised (39).

Research also suggests that hybrid approaches — combining digital work with non-digital teaching, and pairing learners to encourage discussion and collaboration — are more effective than technology use alone. Crucially, impactful use of digital tools depends on substantial teacher professional development, enabling teachers to make informed pedagogical decisions about how, when and why such tools are used to support algebraic learning (36).

Expansion of pre-university algebraic needs

There is increasing recognition of the need for greater attention to data science, computational thinking, modelling and simulation within the intended curriculum (35,36), each of which brings new opportunities and demands for algebra education. Computational thinking, for example, involves skills such as recognising patterns, designing and using abstraction, decomposing problems, deciding when and how computing tools might be used, and defining algorithms as part of a structured solution process. These practices rely heavily on generalisation, representation, and reasoning about relationships, and are therefore deeply algebraic in nature (35).

Programming and algebraic thinking

Evidence from Sweden, where coding is embedded within the algebra curriculum, illustrates the rich educational opportunities that arise from connecting mathematics and programming. Coding activities can often stimulate algebraic thinking; however, differences between the two domains — particularly in their representational systems and syntactic rules — also present challenges. There is a risk that learners' attention may shift away from algebraic ideas towards programming syntax, and research suggests that neither subject is well served when the primary focus is on the other (35).

For example, programming environments such as Scratch can support algebraic learning, depending on how tasks are framed. Scratch representations can help learners explore variable behaviour and relationships, but this potential is only realised when teachers deliberately design and adapt tasks so that algebraic reasoning, rather than coding syntax, drives the learning (45).

More broadly, there is a need for a substantially larger proportion of the UK population to be both confident and competent in mathematics, including algebra, in order to support individual opportunity and wider societal flourishing (8,46). This need spans all levels of attainment. Work on transitions into higher education, for example (47), points to the growing importance of secure and flexible understanding of more advanced school algebra in expanding fields such as informatics, software engineering, computer science and machine learning.

At the same time, there is an equally pressing need for confidence and sense-making in more basic algebra across less mathematically specialised fields and in everyday adult life (8). As increasing numbers of domains become mathematised — often in conjunction with digital developments — a significant proportion of learners will need to remain actively engaged with mathematics, including algebraic thinking, throughout their education. Approaches to supporting this sustained engagement are discussed further in Section 5.

Recommendations

- *Policymakers should* ensure all teachers and learners have access to relevant and reliable digital infrastructure
- *Curriculum designers and teachers* should integrate digital and computational ideas, and use of appropriate digital tools, where they genuinely support algebraic understanding.
- *Teacher educators* should support teachers to make informed decisions about where and how digital technologies may enhance, complement, or replace mechanical/procedural work related to algebra.

4. Evidence of current algebraic capabilities in the UK: performance, foundations and affect

Since the 1997 report, education in the UK has been devolved to the four nations, and curriculum systems have diverged. Participation in algebra education post-16 remains low by International standards in England, Wales and Northern Ireland, yet as argued in Section 3, needs to expand and deepen at every level of algebraic sophistication, and across the UK, to meet emerging needs for individual and national thriving. In UK national assessments (such as the end of Key Stage 2 tests in England, National 5 Mathematics in Scotland, GCSE Mathematics in England, Wales and Northern Ireland, and Higher Mathematics in Scotland), performance is not reported by individual content areas such as algebra, although Key Stage 2 assessments do distinguish between arithmetic and reasoning. By contrast, international large-scale studies such as TIMSS and PISA report performance by both content and cognitive domains, and also gather information on learners' attitudes towards mathematics and their educational and occupational aspirations.

Evidence from TIMSS[1]

There were 12 litres of water in the tank.

Ravi then poured 3 litres of water into the tank and Indira poured another 3 litres of water into the tank.

How can the amount of water in the tank be calculated?

A: 12 + (2 + 3)

B: (12 + 3) + (12 + 3)

C: (12 + 2) x 3

D: 12 + (2 x 3)

Figure 7: TIMSS 2019 Grade 4 item probing structural grasp of number. Source: https://timss2019.org/reports/achievement/index.html#math-4

TIMSS assesses curriculum-aligned performance in mathematics and science at Grades 4 and 8 (ages 9–10 and 13–14, respectively) every four years, most recently in 2023. At Grade 4, there is no separate algebra content domain; instead, the assessment probes structural aspects of number (see e.g. Figure 7). At Grade 8 (Year 9), England's average performance in algebra in TIMSS 2023 was in line with its overall mathematics performance, ranking sixth behind a small group of historically high-performing jurisdictions, though still some distance below them. Overall performance at Grade 4 (Year 5) was also relatively strong (48). No other UK jurisdictions participated.

Analysis by cognitive domain shows a complementary pattern. At Year 5, England's average TIMSS 2023 score was dominated by items classified as 'knowing', with less emphasis on 'applying' or 'reasoning'. At Year 9, performance was dominated by 'knowing' and 'applying', rather than 'reasoning', suggesting that mathematical reasoning — which underpins formal proof and more advanced algebraic thinking — remains an area for further development in many of England's classrooms (48).

Across both grades, England's TIMSS results continue to show a relatively wide socio-economic gap. In 2023, there was also a particularly large gender gap at both levels, evident across content and cognitive domains, as well as in learners' aspirations for further study and mathematics-related careers; this pattern was not seen in most of the highest-performing jurisdictions. Learners' reported confidence, self-efficacy and enjoyment of mathematics, along with aspirations for future study and work involving

[1] https://www.iea.nl/studies/iea/timss

mathematics, were low in both cohorts and, on average, significantly lower than in 2019 (49). These findings raise important and concerning questions about future participation and progression in mathematics, including algebra, in England.

Evidence from PISA[2]

PISA assesses the mathematical literacy of 15-year-olds, with PISA 2022 performance again led by a number of East Asian jurisdictions. Average performance in England was below these highest-performing systems but remained relatively strong by international standards, while performance in the other UK nations was somewhat lower, though still similar to the OECD average. Across all four UK countries, algebra performance — represented by the *change and relationships* content domain — was broadly in line with overall mathematics performance.

Analysis by cognitive domain shows that outcomes related to *reasoning*, *employing* and *interpreting* mathematics were stronger than those related to *formulating*, a pattern with implications for future algebraic functioning and for learners' ability to translate situations into mathematical form. PISA 2022 also placed a particular emphasis on mathematics, providing a detailed picture of learner performance, attitudes and dispositions in this area (50–53).

Recent evidence of UK algebra-specific difficulties

Although there is limited large-scale evidence that reports algebra performance directly, a range of smaller-scale data sources are informative, and wider international findings are also likely to be relevant to the UK. Recent National, Higher and Advanced Higher reports in Scotland, alongside GCSE and A-level examiner reports from awarding organisations in England, Wales and Northern Ireland, point to the persistence of long-standing algebraic difficulties. These include misuse of the equals sign, premature 'closure' of expressions (for example rewriting $3x + 7$ as $10x$ or 10), inappropriate cancelling in algebraic fractions, and errors with indices (such as treating $(x^2)^3$ as x^5 or x^9). Such errors suggest that learners are often not making sense of the algebraic relationships expressed in the symbols they use. Examiner reports also indicate that learners frequently apply the most recently taught technique, rather than interpreting an algebraic statement in terms of the relationships it represents (3). This pattern points to a continued emphasis on procedure over meaning, with implications for learners' longer-term algebraic understanding.

Such errors are consistent with long-standing findings in the research literature (3,4,23) and are particularly concerning in light of the arguments set out in Section 3 about the growing need for secure conceptual understanding in mathematics. Evidence suggests that approaches which deliberately surface and explore conflict — for example, by substituting values into expressions or equations to test their meaning — are more effective for learning than simply restating rules or procedures (3).

Variables and equations can take on multiple related meanings and interpretations in different contexts — a feature often described as *polysemy* — and this is a significant source of confusion for many learners. To address this, learners need experience with a wide range of algebraic actions, and opportunities to distinguish explicitly between different algebraic objects such as formulae, identities, and equations (including equations of lines, functional equations, and equations to be solved) (3).

[2] https://www.oecd.org/en/about/programmes/pisa.html

A particular point of difficulty arises when learners are required to operate on parameters. For example:

For what values of the parameters p and q does the equation

$$(p+q)x^2 + x = 5x^2 + (2p-q)x$$

hold true for every value of x?

Source: Authors

In problems of this kind, learners are required to make two significant conceptual shifts. The first is from reading algebraic expressions as sequences of operations to be carried out, towards interpreting them as representations of relationships between variables. The second is recognising that parameters are not simply unknown numbers to be found, but variables in their own right, whose values determine the behaviour of the expression or equation (3).

Challenges to high-quality algebra learning in the UK are therefore broadly consistent with patterns identified in the wider research literature. There are already challenges in algebraic preparedness for higher education, employment, and personal adult life, with implications for participation, progression and confidence spanning a wide range of levels of algebraic functioning (8,47). Section 3 points also to developing needs. In parallel, evidence from large-scale studies such as PISA and TIMSS, alongside other sources, highlights the importance of fostering productive learner dispositions that support mathematical meaning-making and confidence to continue participation in mathematical thinking. The systems required to support these aspirations more effectively are considered further in Section 5.

5. UK curriculum systems support for desirable pre-university algebraic outcomes

By *curriculum system* we refer to the full range of interconnected components that shape learners' planned experiences in school or college. This includes intended learning goals, subject content, pedagogical approaches and curriculum materials, as well as assessment practices and arrangements for teacher education and professional preparation. It also encompasses wider features of learners' educational environments that influence how the curriculum is enacted.

There is extensive evidence that effective mathematics education depends on a *coherent* curriculum system, in which these components are aligned with one another and with stated policy intentions. Attention therefore needs to be paid not only to individual elements, but to how the system operates as a whole. In this section, we focus in particular on algebra-specific aspects of curriculum, pedagogy and curriculum materials, assessment, and teacher initial and continuing education, while recognising that other systemic factors — such as teacher recruitment and retention — also play an important role.

Evidence reviewed in Section 4 indicates that many learners currently struggle to make sense of much of the formal algebra they encounter, and fail to develop positive dispositions. As a result, they are often not well equipped to use algebra confidently for personal, educational, employment or wider societal purposes. While all elements of current UK algebra curricula remain important for at least some learners, there is limited value in learners spending substantial time developing procedural skills that lack meaning for them. For many learners, this at present includes a significant proportion of the algebraic manipulation they encounter.

Taken together with the changing demands outlined in Section 3, this suggests a clear need for substantial reconsideration of key aspects of UK curriculum systems. In particular, greater attention is required to alignment between curriculum content, pedagogy, and intended outcomes, so that algebra learning supports understanding, confidence and flexible application rather than procedural performance alone.

Curriculum

Current written intended algebra curricula in the UK place a strong emphasis on content coverage and procedural technique, with comparatively less attention given to processes and relationships, although this balance is somewhat different within Scottish curricula.

As set out earlier, there are risks associated with exposing learners to formal symbolic algebra before they have developed a secure base of early and informal pre-algebraic experiences, though these will then productively continue to develop in parallel. Such foundations in relation to current curricula include understanding structures and relationships in number, spatial reasoning and measure, familiarity with number processes and notation, and opportunities to engage in informal algebraic thinking using a range of representations. For these reasons, it may be appropriate for formal algebra to be encountered primarily in post-primary education, while still enabling younger learners to engage in informal algebraic talk and notation, as illustrated in Figure 2, above.

At present, computational concepts and tools remain largely absent from enacted UK curricula, and clearer decisions are needed about how computational education should be organised and integrated.

Similarly, problem-solving and the application of mathematics in meaningful contexts receive limited emphasis, with notable exceptions including Scotland's *Higher Applications of Mathematics* and England's post-16 *Core Mathematics* qualifications.

The Royal Society's *Mathematical Futures* work proposes an enhanced *mathematics and data education* curriculum structured around three strands: foundational and advanced mathematics (re-organised from current provision), general quantitative literacy — including the mathematisation and communication of results — and domain-specific competencies (8). Algebra plays an important role within all three strands, though to differing extents. Achieving a confident and connected grasp of algebraic relationships and representations within *foundational mathematics* would likely require relocating a substantial proportion of algebra content to *advanced mathematics*, while ensuring that this remains accessible to a significant proportion of each cohort.

Pedagogy and curriculum materials

The curriculum that learners experience in classrooms is strongly shaped by the curriculum materials in use, including physical or digital textbooks, workbooks, online resources, and physical or digital manipulatives. Teachers engage with these resources in different ways, adapting and mediating them to meet local needs. There is strong evidence that when curriculum materials are used in ways that align with intended goals — such as targeting development of a secure conceptual understanding of algebraic ideas — they can support improvements in learners' algebraic thinking (54,55). At the same time, research also suggests that many commonly used materials place a greater emphasis on routine algebraic procedures than on developing deep conceptual understanding and meaning-making (56).

In light of this, classroom experiences would benefit from rebalancing, so that alongside procedural fluency they more deliberately support productive learner dispositions towards mathematics, as well as the development of self-regulation and metacognitive awareness.

Assessment

The 1997 Report says (p3): *'Current assessment practices in mathematics tend to place more emphasis on correct answers than on the process of solution'*. Our high-level review of recent UK mathematics assessments at age 16 (GCSE/N5) and the associated mark schemes suggests that at present, reasonable credit is often given to procedure, but comparatively little attention is paid to assessing or crediting underlying conceptual grasp, meaning-making or genuine algebraic reasoning. For example, learners are only infrequently asked to create an algebraic equation to model a given situation, or to critique an algebraic argument, even though such learning is targeted by national curricula. When assessments are perceived as high-stakes, teachers understandably prioritise aspects of teaching that appear to be rewarded by those assessments (57).

The Royal Society (8) argues that, rather than expecting summative assessment to serve multiple purposes such as selection and standards monitoring, greater priority should be given to assessment approaches that recognise and communicate what learners genuinely know and can do. This requires strong alignment between assessment methods — both tasks and mark schemes — and valued learning outcomes.

While the capacity for digital (formative and summative) assessment continues to grow, its impact, as noted in Section 3, needs careful monitoring to identify any unintended consequences. In particular, the automated analysis of learner work remains highly challenging in relation to complex learning outcomes, such as problem-solving, or algebraic literacy, reasoning, or proof. For the foreseeable future, fully automated approaches are unlikely to be able to capture the full range of algebraic activity and understanding that curricula seek to promote.

This creates an ongoing challenge in determining where and how human judgement is most appropriately involved in assessment processes. The Royal Society (8) argues for the development of online summative assessment approaches that allow for meaningful human input where needed, while also being capable of evolving and scaling over time as technologies and assessment practices continue to develop.

Teacher initial and continuing education

Moving towards the approach to algebra teaching and learning outlined above would place substantial demands on teachers. The aim is not for learners to study less algebra, but to learn it more securely, confidently and meaningfully, in ways that encourage continued engagement and support mathematical thriving in an increasingly digital and mathematically rich world. Achieving this would require sustained commitment from educators across all phases, from the early years onwards, and significant investment in both beginning and experienced teachers developing the necessary knowledge and skills (35,36).

Evidence from previous mathematics curriculum reforms suggests that such change is neither quick nor straightforward (35). Efforts to improve algebra teaching and learning are therefore unlikely to succeed without addressing wider systemic challenges, including those in recruiting and retaining secondary mathematics teachers (58,59), and in strengthening the mathematical preparation and support of the early years workforce (12,20).

Recommendations:

Curriculum and assessment designers should:

- Rebalance meaning-making and procedural knowledge within algebra curricula, so that they fully support the recommended aspirations for teaching. This may require careful consideration of sequencing and curriculum coverage.
- This may require careful consideration of sequencing and curriculum coverage.
- Ensure assessment, including both tasks and mark schemes, fully reflect and value this balance, and includes authentic use of a small range of selected digital tools.
- Monitor digital assessment of algebra carefully for validity and unintended consequences, including where limitations of digital assessment may serve to narrow the taught curriculum.

Teacher education and system capacity policymakers should:

- Invest in initial and continuing teacher professional development to support rich and digitally-informed algebra teaching coherent with such a rebalanced, meaning-making algebra curriculum.
- Address systemic constraints, including teacher recruitment and retention.

6. Reflections on the 1997 Algebra Report conclusions

While almost all of the conclusions reached in the 1997 report (1) remain valid, few — if any — have yet been fully addressed. Since then, however, a substantial body of additional evidence has emerged. This strengthens understanding of the central role of sense-making in successful algebra learning, the importance of early mathematical experiences in shaping later algebraic understanding, the relational foundations that grow out of number, and the critical need to support learners' metacognition, self-regulation and positive affect. In reflecting on the conclusions of (1), we therefore note the following:

- Except in Scotland, the UK algebra curriculum and accountability 'pendulum' has swung back to privilege algebraic procedures, often marginalising education for grasp of structure and meaning: there is good evidence to support enhancement of the latter, to support robust and confident grasp of algebraic thinking at much greater scale.
- Recent, as well as older, work, suggests we should better value informal and pre-algebraic experiences from early years onwards, as well as the development of learners' metacognition, self-regulation and positive affect in relation to the learning of algebra.
- The world of 2026 is more digitally-pervaded than it was in 1997, bringing more widespread algebraic needs for personal and societal thriving, yet our UK algebra curriculum systems have not yet evolved to respond appropriately to those changes.
- Digital tools are now more widely-available, and further developed, than in1997. There is good evidence that curriculum, pedagogy and assessment should each draw on the appropriate use of a limited range of digital tools for the teaching and learning of algebra.
- The current review therefore suggests a re-shaping of the curriculum in order to support the development of a curriculum system that has strengthened coherence, especially in relation to assessment.
- Post-1997 evidence continues to suggest that good teaching and learning of algebra is a complex and demanding endeavour. Significant improvement requires concerted efforts from policymakers, teacher educators, curriculum and assessment designers, and teachers - as reflected in the Recommendations made.

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Appendix 1: approach to review

Key features of design of this review were the scope and audience intended. We took the scope to be the teaching and learning of algebra and its preceding foundations pre-university, where ‘algebra’ is understood as in Section 1, and excluding calculus, which, while rich in the use of algebra, we conceptualise as school-level mathematical analysis. The focus was on what is new, or of renewed focus, in this field since the Royal Society/JMC Teaching and Learning of Algebra Report of 1997 (1). The intended audience was clarified to be teachers of mathematics (and especially, of course, of algebra) and other mathematics education stakeholders (including mathematics education policymakers and policy influencers), many of whom lie outside the mathematics education research community.

We then began from a corpus of post-1997 major overview and systematic review publications, focused on the teaching and/or learning of either algebra or wider mathematics (2–5,8,16,20,21,32,35,41); also recent JMC reports, which have a specific focus on work in the UK. Those were analysed for key themes under the identified headings/Sections of the Review. From those sources we identified key individual citing or cited papers (the balance of those referenced), and analysed those separately to allow greater critique, depth, and nuance. We complemented those with scrutiny of a high-level review of algebra-focused papers published only very recently (2021 onwards). The final report, though, represents a significant contraction of the knowledge represented in the consulted material, interpreted for the intended audience, and there is no claim the overview presented is totally systematic or unbiased.

Appendix 2: Key ideas and experiences

Some examples of key algebraic ideas and activities taken from each stage, intended to be illustrative. The selected examples are neither comprehensive nor prescriptive.

Later stages should aim to incorporate and continue to develop deeper and richer grasp of elements introduced informally at earlier stages.

	Examples of key ideas
The early building blocks for algebra, occurring across children's experiences.	• Balance • Simple Covariation • In betweenness • Relative size • Close/far • Pattern
The wider mathematical context for ages 3-9	• One-to-one and one-to-many correspondence • Additive and multiplicative relationships • Relational meaning of the equals sign • Ordering and comparing • Informal ideas of measures • Representing patterns and structures • Exploring numerical structures • Finding mathematical structure in everyday activities (e.g. books, puzzles, rhymes, games) • Making connections between numbers, quantities, and relations
Developing algebraic ideas through ages 7-14	• Ordering • Balance and imbalance • Equivalence • Operations and their inverses • Symbols can represent different types of quantity • Creating and refining representations of numbers, patterns, and relationships • Recognising and describing patterns • (Informally) describing and generalising relationships • Making connections between arithmetic and algebraic reasoning (inc. commutativity, associativity) • Selecting and critiquing strategies
Algebraic experiences through to pre-university	• Freedom and constraint • Invariance and change • Doing and undoing • Characterising and organising • Mathematising and modelling • Equality and inequality • Functions, building to a wide range of representations as in Section 2. • Affordances and constraints of different representations • Selecting appropriate representations from a wider range • Representing patterns and relationships with symbols in normative ways