JMC
Joint Mathematical Council of the UK

Mathematics education and digital technology

A report from a working group of the

Joint Mathematical Council of the United Kingdom

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July 2023

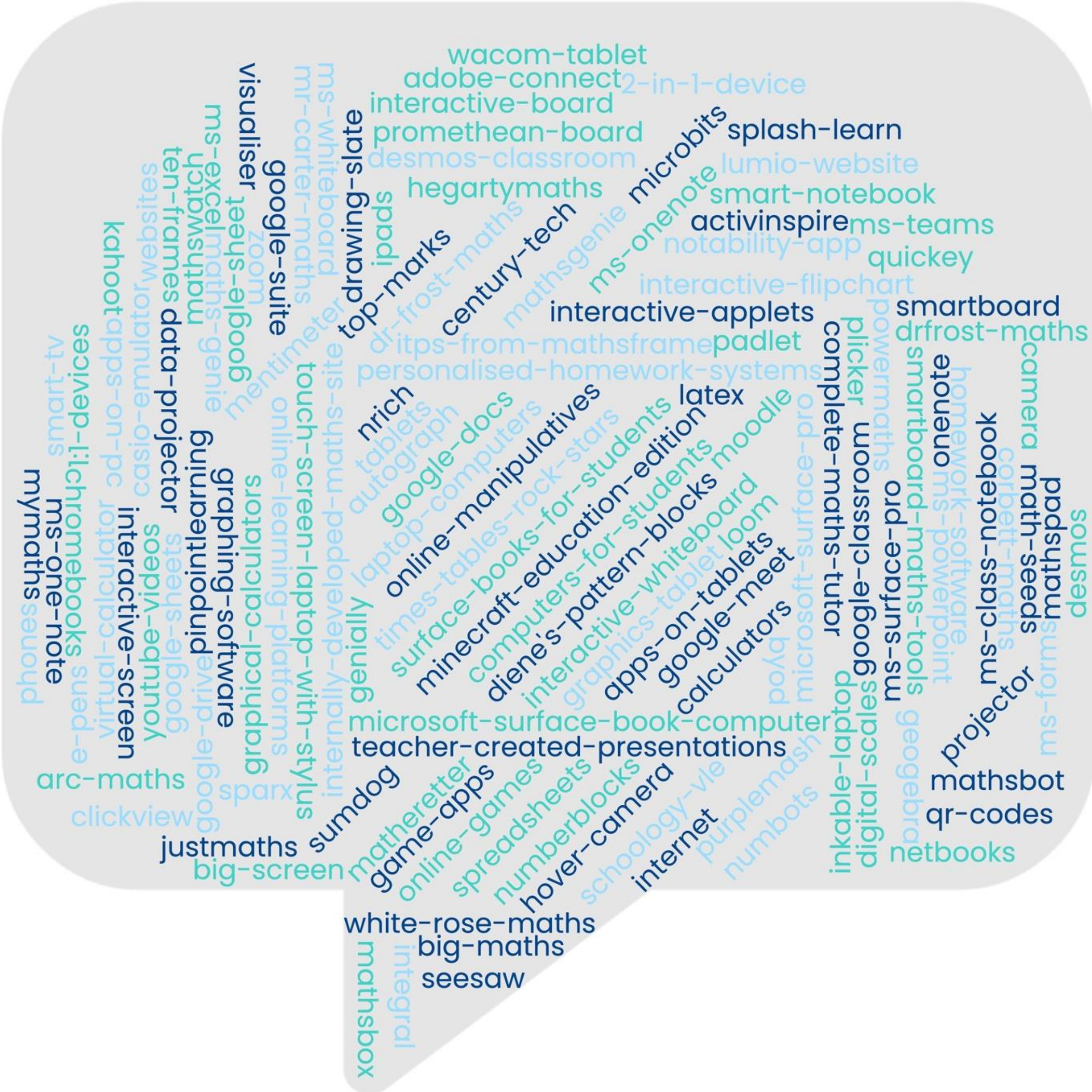

Figure 1: word cloud based on 395 technologies that respondents used and considered to be impactful

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Executive summary

In September 2011, the Joint Mathematical Council of the UK published a report on Digital Technologies and Mathematics Education. Ten years later, the JMC Trustees committed to revisiting and updating this work in the light of changes in education systems, transformations in the availability and uses of digital technologies, and following the impact of the global pandemic.

This 2023 Report, and the JMC Working Group's activity that preceded it, was motivated by a strong sense that despite the promises of digital technology to enhance mathematics education, and the ongoing transformation of all aspects of modern society by technology, little has changed in the intervening years since the publication of the 2011 Report. Progress against its recommendations has been slow at best.

There has been considerable change in the use and application of digital technologies in all aspects of the education system, but when it comes to mathematics education this has been rather organic, patchy, inequitable and with little evidence of widespread impact. That said, in HEIs, there has been a rapid shift in the use of digital technologies and genuine efforts to use the tools that professional mathematicians and users of mathematics will use in their employment futures.

This leaves a considerable gap between the use of digital technologies in school curricula and assessment and the kinds of uses that are made of these in other subjects and in the jump from compulsory education to higher education and work. In Scotland, the new Higher in Applications of Mathematics is demonstrating that change is possible and that assessments need to value the kinds of technology used in enacted curricula in classrooms.

The challenges facing the UK nations in adopting digital technologies effectively into the teaching, learning and assessment of mathematics are not unique. Analysis of recent change agendas in other countries highlights considerable barriers to achieving systemic change, but that change is possible under the right conditions. Unfortunately, the state of implementation science, including our understanding of how sustained and effective change strategies in complex education systems can be designed, implemented, and evaluated is underdeveloped.

This new Report presents a high-level synthesis of a working group of the Joint Mathematical Council which was convened through 2022/3. The landscape of digital technology in mathematics education is complex and the Report retains a 'helicopter view'. Acknowledging that major systemic change is difficult to achieve, we focus on relatively small, achievable recommendations which, if done well, could amount to significant progress in an area where change has been difficult to achieve.

The sections of the report can be considered as three scales, yet each addresses similar issues and challenges. They explore aspects of technology use:

- in classrooms and schools (Section 2).
- nationally, in terms of assessment (Section 3).
- in other nations (Section 4).

Tentative recommendations from each of these strands of the Working Group's activities are synthesised in Section 5 into the following twelve recommendations, organised into four themes.

Recommendations on curriculum reform

1. **Future reforms of statutory curricula** or non-statutory guidance should explicitly address the use of digital technology in the teaching of mathematics and data analysis across all years. Well-designed support for the implementation of reforms should also be provided.
2. **Computational tools should be actively encouraged** (e.g., digital and physical calculators, spreadsheets) in primary and secondary classrooms to enhance pupils' exploration and understanding of number, and to complement their fluency with mental and written methods of calculation.
3. **Universities' use of digital technologies** in mathematics and applied-mathematics courses should inform the innovation and design of pilot post-16 mathematics programmes.

Recommendations on assessment

4. **Appropriate high-stakes assessment of digital technology skills** is needed to ensure that such skills are embedded in, and developed throughout, the learning process.
5. As online assessment develops, the **opportunity to embed mathematical digital technologies must be taken early**. This should include the use of computational and graphing tools, and dynamic geometry tools as appropriate. This will require significant development work by awarding organisations and regulators.
6. **National bodies should ensure that skills for working with data are better assessed**, through ongoing practical work or high-stakes examinations in which students have access to appropriate digital technologies.

Recommendations on professional development

7. All **professional development for teachers of mathematics should embed appropriate uses of digital technologies** such that they become normalised, rather in the same way that digital technologies should be embedded in all teaching and learning.
8. **Professional development programmes on digital technology** should focus on the effective use of digital technologies that teachers (and learners) use frequently in order to increase the likelihood of widespread adoption.
9. **A cadre of digital technology 'champions' for mathematics** should be developed, who a) are knowledgeable about digital technologies for mathematics education, b) can support sustainable change management in a rapidly evolving educational technology landscape, and c) can design and deliver professional high-quality initiatives for teachers of mathematics.

Recommendations on implementation, resourcing and leadership

10. **Common access to a *limited* and *agreed* set of phase-appropriate digital technologies** and associated professional and technical support is needed to ensure system-wide equity and coherence for all practitioners and their learners.
11. **Support for leadership teams to develop appropriate digital strategies** for mathematics is needed. This must ensure sustainable access to both the technology and the associated technical, pedagogic, and curricular (institution-based) support for practitioners.

12. **High-quality formative evaluation** of any large-scale intervention using digital technologies for mathematics education is essential and should be included in the implementation design from the outset.

The promises of digital technologies for enhancing varied aspects of mathematics teaching and learning, from reception to undergraduate study, have yet to be fully realised. Indeed, given the rapid pace of technological change and the slow lead time for designing and implementing systemic change in the education system, there is every possibility that this will continue to be the case.

We hope that the findings from the Working Group's activities and the above recommendations can further contribute to the debates, and influence leadership and policymaking activity in mathematics and related areas. We believe that the recommendations could have considerable impact, particularly if implemented in a well-orchestrated way.

Arguments about the value of mathematics are well-made and widely accepted. It is now time to ensure that the digital tools that are now commonplace, both for doing mathematics and for applying mathematical and data ideas in a range of education, work and life contexts, are widely and equitably adopted to enhance mathematical attainment and outcomes for all.

July 2023

About the JMC

The Joint Mathematical Council comprises many participating and observing member bodies from across the UK. Their complementary expertise covers the teaching and learning of mathematics in schools, colleges and universities; the initial training and ongoing professional learning of teachers and lecturers; mathematics education research; educational policy and mathematical applications and interests more generally.

The JMC makes representations to government and other bodies both proactively and reactively. It works closely with the Royal Society's Advisory Committee on Mathematics Education, oversees the British Congress of Mathematics Education and undertakes other targeted projects and activities, producing occasional reports and working papers.

1. Introduction

In September 2011, the Joint Mathematical Council of the UK (JMC) produced a working group report on Digital Technologies and Mathematics Education. Ten years later, the JMC Trustees committed to revisiting and updating this work in the light of changes in education systems, transformations in the availability and uses of digital technologies, and following the impact of the global pandemic.

The 2011 Report showed that the potential for digital technologies to enhance mathematics education in the UK had not been realised and so it recommended changes to curriculum and assessment as well as increased professional development opportunities to better exploit this potential. Unfortunately, relatively limited progress has been made against the 2011 Report's recommendations. Furthermore, with the fragmentation of parts of the schooling system - in England at least - approaches to digital technology use have become more diversified and coordinated systemic change more challenging.

That said, a great deal has changed and continues to do so. New global economic, climate and health challenges are accompanied by accelerated technological innovations and growing concerns over equity, all of which require educationalists to consider how they can support learners[1] to navigate a constantly changing world that is mathematically formatted, data saturated and digitally networked.

Digital technologies are now embedded in educational processes and the Covid-19 pandemic resulted in a rapid transition to blended and online learning with a consequent influence on mathematics educators, pedagogic approaches and students' learning. Alongside this, mathematics and data science are becoming increasingly important in most aspects of cultural, civic, commercial, health and working life. Mathematics education is therefore under pressure to evolve so that learners can develop the new skills, knowledge and understandings to thrive in contemporary and future workplaces and society.

Over the past decade several educational systems around the world have attempted to address some of these challenges at a policy- or system-level through initiatives that aim to integrate digital technologies in mathematics or in closely related areas of the curriculum. Such initiatives include the foregrounding of computational thinking, using online learning environments, and the expansion of technology-mediated assessment. The JMC was keen to understand some of the international developments in this area, and the extent to which they show promise, either in better exploiting the potential of digital technologies or in laying the groundwork for doing so. The country case vignettes in Section 4 offer some examples.

Elsewhere in the report, the role of calculating/computational devices and apps, and the extent to which their contested use has changed over time, is considered. Similarly, data generated for the report included examples from the past where high expectations of students' use of graphing calculators, for example within the SMP 16-19 A level and Nuffield Advanced Mathematics schemes, have largely disappeared. In this respect, there appears to have been a step backwards. There are of course always exemplary practices in any area of education, but it seems that in schools at least, making the most of digital technologies for mathematics education is hard to achieve at scale.

[1] The Report uses students, learners, and pupils variously. The Working Group did not think that any one of these terms adequality covered all of the contexts under discussion.

This is a timely moment to be revisiting this problem, given the Prime Minister's 2023 ambition for a mathematically well-educated and digitally literate workforce. His primary motivation is to improve the economy, yet the gap between the uses of technology to do mathematics in the workplace and its uses in schools is considerable and longstanding[2]. Any approach to improving mathematics must also take account of the uses of technology for mathematical and data science applications. In contrast to schools, the commitment to technologically enriched teaching and assessment has moved apace in universities (see Section 3). In higher education institutions (HEIs), the lack of need for national assessment standardisation, and the imperative of the employability agenda has provided a more conducive environment for innovation and progress. As a result, the digital divide from school to university is now becoming greater, whilst the chasm between technology use pre- and post-18 widens.

The post-pandemic period was considered by some to be the platform for EdTech's 'great leap forward' but that moment seems to have passed. It has been superseded with the promise - or threat, depending on one's perspective - of artificial intelligence (AI) and the cascading updates to natural language processing (NLP) models that power tools such as ChatGPT, Bard and the like. This raises new questions about AI's impact on education and assessment generally, and in specific ways for mathematics education. In one sense, rapid developments in AI have pushed the debate on from recent arguments for a 'computational thinking' (CT) curriculum. Whatever the curriculum of the future looks like, careful consideration of the professional development needs of mathematics educators, and better understanding of impediments to systemic change, are essential.

One of the problems with developing digital technology use is creating the space within the curriculum to innovate. In order to create the time and motivation to introduce and develop new digital skills and competences something needs to give, both within the curriculum and in terms of the dominating influence of high stakes assessment and accountability measures. This question of what the future of mathematics education should look like is being considered by the Royal Society's Mathematical Futures Programme[3]. However, any attempt to predict the future, particularly when it comes to digital technology is almost certainly bound to fail; there is an inherent unpredictability in complex systems, and school education systems have been shown to have considerable inertia.

Bringing about substantive change in education is difficult given the challenge of designing and implementing effective change strategies. This might be even more difficult for digital technology use due to the pace of change, its highly adaptive and organic development, and a lack of scalable support. Similar to spreading the best classroom pedagogy, the problem is not the 'what?' but rather the 'how?'; how do system leaders make it happen at scale. Two relatively recent large-scale initiatives in England, Scratch Maths[4] (for students aged 8-11 years) and Cornerstone Maths[5] (11-14 years) have explored this challenge and offer models for a coordinated approach to topic-based curriculum design,

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[2] Hoyles, C., Wolf, A., Molyneux-Hodgson, S. & Kent, P. (2002) Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council. Institute of Education, University of London; Science, Technology and Mathematics Council: London.

[3] https://royalsociety.org/topics-policy/projects/mathematical-futures/

[4] https://www.ucl.ac.uk/ioe/research/projects/ucl-scratchmaths

[5] https://www.ucl.ac.uk/ioe/research/projects/cornerstone-maths

associated assessment and teacher professional learning, though neither have had widespread influence[6,7].

About the working group and report

A JMC discussion paper[8] was presented to the June meeting of the JMC[9] in 2021 and provided the foundation for the Working Group (WG or 'the Group'). Members of the WG were recruited later that year, each coming from one or more of the JMC's 'participating bodies' with the Group holding its first monthly meeting in January 2022, online.

An initial challenge for the WG was finalising and agreeing the scope for its activity. Given the huge diversity of digital technologies in use, the Group had to decide how best to focus its limited resources. Various categorisations of digital technologies exist. For example, a recent evidence review[10] of technology for improving learning considered the following technology types based on its functional pedagogic usage: those for drill and practice, simulations, mobile learning, game-based learning and pedagogical scaffolding or intelligent tutoring system.

In general terms, one can distinguish between digital tools for doing mathematics and digital tools to support teaching and learning. The latter group includes generic/non-mathematics specific digital tools, such as Virtual Learning Environments (VLEs) and online applications for synchronous (live) teaching (i.e., Zoom), but also those that, understandably, use the functionality of digital tools for doing mathematics. For example, some digital resources (e.g., Desmos Classroom) are built around more mathematical applications - especially graphing, geometry and data handling tools - that require learners and teachers to actively engage with the tools. Other applications provide less interactive content but support consolidation, practice, and assessment tasks. In both cases, the applications can be aimed at whole-class, small-group or individual learning contexts.

Given the large number and diversity of digital applications and tools available, the WG avoided attempts to classify these from the outset, but used the responses from a UK-wide survey of practitioners in schools and colleges (see Section 2) to produce the following loose classification:

- **Technology devices**: e.g., visualisers, interactive whiteboards, iPads/Laptops/Chromebooks, graphics tablets and display devices.
- **Mathematical applications and tools**: e.g., Desmos, GeoGebra, Autograph, spreadsheets, calculators (including graphical calculators).
- **Mathematical content platforms and resources**: e.g Sparx Maths, Khan Academy.

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[6] https://educationendowmentfoundation.org.uk/projects-and-evaluation/projects/scratch-maths
[7] Clark-Wilson, A., Hoyles, C., Noss, R. et al. Scaling a technology-based innovation: windows on the evolution of mathematics teachers' practices. ZDM Mathematics Education 47, 79–92 (2015). https://doi.org/10.1007/s11858-014-0635-6
[8] Golding, J. & Lyakova, S. (2021) School mathematics education and digital technologies: discussion paper for JMC. https://www.jmc.org.uk/wordpress-cms/wp-content/uploads/2021/11/Digital-tools-for-the-teaching-and-learning-of-mathematics-FINAL-070521.pdf
[9] The JMC & RS/ACME also co-hosted a Mathematics Education and Digital Technologies workshop in May 2011
[10] EEF, 2019 https://educationendowmentfoundation.org.uk/education-evidence/guidance-reports/digital

- **Programming languages**: Python, R, Matlab, Scratch, etc.
- **Generic educational applications**: Kahoot, Padlet, Loom, etc.
- **Generic technology applications**: word processers, video conferencing software, etc.
- **Professional community platforms**: Hwb (Wales), GLOW (Scotland), NCETM, etc.

In addition to surveying practitioners, the Group agreed that it would be important to consider the role of digital technology in high stakes assessment for three reasons: (1) it is widely understood that assessment shapes the enacted curriculum according to the adage 'what you test is what you get' (WYTIWYG[11]), (2) the recent experience of the introduction of the large dataset task in the reformed A level (England) when compared to the new Applications of Mathematics (Scotland), and (3) the growing interest in developing online, high-stakes assessments following the disruptive impact of the pandemic on traditional assessment systems. The impact of assessment on the use of technology has been considered at four different levels of education in the UK.

In addition to regular, lively discussions of the evidence in the extant research and grey literatures, the WG engaged with a range of key stakeholders in the education sector, including education businesses, industry, civil servants, and practitioners. The JMC also commissioned a rapid review of evidence of international examples where digital technologies have been adopted and the associated affordances and constraints.

The Group agreed to retain a broad view of digital technology and mathematics education across phases, from primary to higher education, and from across the UK. The composition of the Group reflects this (see Appendix) and activities were designed in ways that included phase and regional perspectives. The Group brought together considerable expertise and experience, including experts in digital technologies and those familiar with the challenges of implementing educational change at different scales.

This report presents a high-level synthesis of the Group's work. The landscape of digital technology in mathematics education is complex and we try to retain a 'helicopter view' throughout, offering a small number of concluding recommendations. Acknowledging that major change is difficult to achieve, we focus on small changes which, if done well and widespread, would amount to significant progress in an area where change has been difficult to achieve.

The sections of the report can be considered as three scales, yet they all cover similar issues and challenges. They explore aspects of technology use:

- In classrooms and schools (Section 2)
- Nationally, in terms of assessment (Section 3)
- From other nations – what can we learn? (Section 4)

Each of the Sections generated pertinent recommendations. In the concluding section these are synthesised as a single set of twelve *key recommendations*.

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[11] Burkhardt, H., & Swan, M. (2012). Designing assessment of performance in mathematics. *Educational Designer*, *2*(5), 1-40.

2. Digital technology use in formal and informal educational settings

In mathematics education, a range of factors influence whether, and how, teachers use technology in their teaching[12] including "their beliefs about, and attitudes towards the technology, as well as their perception of the nature of mathematical knowledge and how it should be learned" (p.1). Alongside these, other major obstacles such as lack of time, limited professional learning opportunities and poor access to the technology and necessary technical support seem to prevail across most education systems. Recent studies into online learning during the pandemic by the Scottish[13] and Welsh[14] governments, and by Ofqual[15], highlighted a digital divide in terms of participants' access to devices, quality of internet access and learning loss.

One of the priorities for the Working Group was to gather evidence from practitioners across the four nations of the UK, and from all phases of compulsory education to shed light on current digital technology use in formal and informal settings for the purpose of teaching, learning and assessing mathematics. In order to do this, a national survey of practitioners was undertaken in the summer of 2022 and, thereafter, online interviews with curriculum leaders were conducted.

Survey of UK practitioners

The survey of practitioners' perceptions and uses of digital technologies to support mathematics education aimed to understand practitioners':

1. **uses of technologies** within the planning, teaching and assessment of mathematics: which digital technologies they value the most, how they use them, and what they think they (and their learners) gain from their use.
2. **views of the barriers** that prevent them from developing their use of digital technologies.
3. **professional learning needs** in different phases.
4. **perceptions of which areas of mathematics** (content and pedagogy) can be supported by using technology alongside their self-assessed competence and confidence.

The survey was aimed at practitioners in both formal and informal settings who had a responsibility for the teaching of mathematics to learners of all ages across the four nations of the UK.

The following definition for digital technologies was used in the survey:

> *all types of digital devices, software, applications, platforms, online resources and tasks that might be used to support the teaching and learning of mathematics.*

However, due to the wide range of internet-based platforms, applications and resource portals that offer multiple access points to the same or similar tools and resources, it was not the purpose of the survey to collect detailed information on the types of technologies being used. To further complicate

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[12] Clark-Wilson, A., Robutti, O. & Thomas, M. Teaching with digital technology. ZDM Mathematics Education 52, 1223–1242 (2020). https://doi.org/10.1007/s11858-020-01196-0

[13] https://www.gov.scot/publications/education-recovery-key-actions-next-steps/

[14] https://research.senedd.wales/research-articles/back-to-school-education-in-the-time-of-covid/

[15] https://dera.ioe.ac.uk/id/eprint/38271/2/6803-2_Learning_during_the_pandemic-_quantifying_lost_time.pdf

matters, there is a wide range of terminology used by both the technology sector and educational practitioners to describe similar digital resources. The focus of the survey was therefore on practitioners' uses of digital technologies, their perceptions of these uses and their associated professional learning experiences and needs. A few of the main findings are summarized herein but a full technical report for the survey can be found on the JMC website.

About the survey and its respondents

The survey was open from 1st June - 17th July 2022 and was aimed at practitioners in both formal and informal settings who had a responsibility for the teaching of mathematics to learners of all ages across the four nations of the UK. Sampling was opportunistic and there were 228 valid responses from practitioners in the four nations who taught mathematics during the 2021-22 academic year. Table 1 presents an overview of the responses.

				Post-16		
	Early years/ Foundation	Primary	Secondary	A level Senior	GCSE resit, Adult learning	***TOTAL***
England	1	21	76	26	10	134
Northern Ireland			2			2
Scotland	1	14	6	13	6	40
Wales	1	3	35	12	1	52
Total	3	38	119	51	17	228

Table 1: Distribution of respondents by the nation and phase in which they taught during the 2021-2 school year.

Uses of technology

The 228 respondents named a total of 395 technologies that they had used and considered to be impactful (see Table 2). Of these, 87 were distinct and the distribution across the categories listed in the Introduction is shown in Table 2.

Type of technology	Technology devices	Mathematical applications and tools	Mathematics content platforms & resources	Generic educational applications	Generic technology applications	Professional community platforms
Total	19	14	19	14	19	2

Table 2: Distribution of technology types reported (n=228).

Respondents were first invited to nominate a technology that they had used in their practice and were then asked to select up to three purposes for its use. These results are summarised in Table 3.

Purpose for the use of technology	Frequency (%)
for presenting mathematics	139 (61%)
to introduce or develop new concepts in maths	130 (57%)
for student practice and consolidation	118 (52%)
to assign and monitor classwork or homework	104 (45%)
to encourage collaboration and/or discussion	66 (29%)
for assessment and feedback	59 (26%)

Table 3: Purposes assigned to the use of technology (n=228).

Given the size and distributions of the sample, there are risks with breaking down this data into further categories[16]. However, the findings do raise questions about the relationship between the different curricula and phases in the four nations and practitioners' purposes for their technology use. For example, the proportion of respondents in Scotland who use technology to support student practice and consolidation is somewhat higher than in both England and Wales (see the Technical Report for further details). Also, in the primary phase there was greater reported use of technology to support collaboration and discussion, when compared to the other phases.

Respondents were then asked how their technology use (as a teacher of mathematics) had changed over the previous 3 years, which included the period before the Covid-19 pandemic.

- 169 (74%) said that their use had increased
- 51 (22%) reported about the same usage
- 3 (1%) reported decreased use

Of those respondents reporting increased use, their reasons are summarised in Table 4 (respondents could select up to three).

Reason for increased use	Freq (%)
Technology for maths is more available and/or easier to use than previously.	76 (45%)
My workplace provides good access to technology for use in maths.	75 (44%)
I am keen and have devoted time to develop my practice.	68 (40%)
I use technology more because of the Covid-19 pandemic.	68 (40%)
Technology motivates my learners to learn maths.	62 (37%)
I better understand how technology can deepen my learners' understanding of maths concepts.	56 (33%)
My workplace provides effective support for me to develop my technology use in maths.	20 (12%)
My line manager expects me to use technology in maths.	18 (11%)

Table 4: Reasons given for increased used of digital technologies (n=169).

[16] Although statistically robust comparisons between the nations and phases are not possible, the accompanying Technical Report does explore respondents' reasons for their choices, which provides valuable contextual data.

Unsurprisingly, two fifths of respondents attributed the increase in use to the Covid-19 pandemic, which probably links to the use of digital applications such as Microsoft Teams, Google Meet and Zoom to support synchronous and asynchronous online learning. It is notable that approaching half of the respondents reported that their workplace provides good access to technology for use in mathematics.

Professional development and digital technology

Practitioners reported the nature of their prior professional learning activities that had supported their use of technology, which is summarised in Table 5 (N.B they could select all options if appropriate).

	With an expert trainer	With colleagues	On my own
Exploring new technologies and digital resources	39 (17.8%)	125 (57.1%)	147 (67.1%)
Participating in formal training on specific technologies	64 (37.4%)	79 (46.2%)	78 (45.6%)
Developing activities that make use of new technologies	14 (7.2%)	95 (48.7%)	143 (73.3%)
Reflecting on activities that use new technologies	9 (4.5%)	98 (49%)	144 (72%)

Table 5: Practitioners professional learning experiences concerning new technologies (n=228).

The proportion of practitioners who engaged in any professional learning experiences with an expert trainer was low. This might be due to a lack of opportunities, or it might be that the respondents are a more expert group of practitioners. Approximately half of the respondents report engagement with colleagues as an impactful mode of professional learning, which might imply that they are engaging in some collaborative planning in relation to technology use in mathematics. Respondents described their most impactful professional learning experiences, which highlighted the need for some level of expert support (often accessed at conferences and formal training programmes), collaborations with colleagues (that also involved a more expert colleague) and, in most responses, working alone.

When asked if they would appreciate more professional learning opportunities, 65% reported they would, 22% said “Maybe” and 13% said “No”[17]. The nature of these desired opportunities range from bespoke in-school training with hands-on support through to more formal and sustained programmes of support that focused on: 1) specific areas of mathematics, 2) particular digital resources and 3) particular pedagogic purposes. Practitioners who responded “No”, gave reasons such as a lack of access to well-maintained technology, or a lack of purpose for its use in mathematics, which again might suggest that clearer leadership strategies for the use of technology in mathematics are needed. It is striking that more than a third of respondents (35%), are either equivocal or negative. Greater provision of professional development opportunities will arguably be insufficient without a more coordinated and strategic approach.

[17] This desire for more PD to support the integration of digital technology into mathematics teaching reflects the findings of TIMSS 2019, see Richardson, M., Isaacs, T., Barnes, I., Swensson, C., Wilkinson, D., Golding, J. (2020). Trends in International Maths and Science Study (TIMSS) 2019 National Report for England. London: DfE.

Looking to the future, practitioners were asked about the areas of mathematics they perceived could be supported using technology, to name a specific technology if they could, and report their associated competence and confidence to use that technology. Under half of the overall survey respondents offered a response, which have been analysed by broad curriculum area and phase in Figure 2 below.

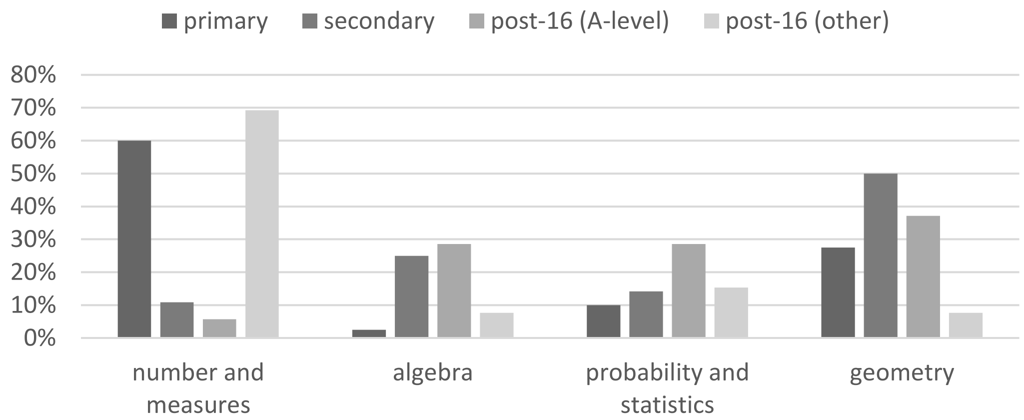

Figure 2: Practitioners' suggestions for curriculum areas by phase (n=102, which is the subset of respondents who named a specific technology that they had actually used).

Although the same caveats regarding the interpretation of the small samples still apply, these findings do indicate some promising areas for future curriculum development and support which are addressed at greater length in the accompanying Technical Report. Most notably, the levels of these practitioners' reported confidence (how self-assured in its use) and competence (how well the different features are used) is variable but does indicate a more confident and competent group who could support other practitioners in the future.

Curriculum leader perspectives

To get a slightly broader view of uses of digital technologies, the survey was complemented by exploratory interviews with ten mathematics leads in primary, secondary and post-16 education from across England, Scotland, and Wales. The interviews explored the general uses of technology in these ten settings and more widely as some mathematics leads had oversight of several schools or colleges. The conversations centred upon the use made of digital technologies in the teaching and learning of mathematics in their setting(s). A wide range of uses of digital technology were reported, dependent upon a variety of factors, although some themes did emerge. The responses are summarised in the following four areas: 1) teachers' use of technology, 2) students' use of technology, 3) common themes, 4) professional learning.

Teachers' use of technology

Teachers across these settings commonly have access to a laptop or PC in their teaching room and a means of projection which may have an interactive element; there is a trend away from interactive boards. These are typically used for presenting mathematics with lessons planned to use PowerPoint, or software specific to their whiteboard. The use of visualisers was referred to in both primary and secondary settings. Teachers related instances where getting the technology set up was time consuming and complicated, and such negatives outweighed perceived benefits.

One respondent described spending a great deal of time creating PowerPoint presentations but that these can be quite rigid, getting “derailed” when children don’t learn in the way the teacher anticipated. One mathematics lead reporting using types of software for lesson planning which enabled their teaching to be more adaptive.

Students’ use of technology

Regular access to laptops or tablets can help to facilitate individualised learning for students. However, student access to hardware in the classroom varies. Tablets are viewed to be logistically more manageable than laptops, but replacement costs can be a challenge; older machines needing to be updated to meet security issues. Students commonly used digital technology to revise or catch-up specific areas of learning.

Beyond the classroom, it was common to hear of online platforms being used for homework, student revision, and other independent learning. This raises an equity issue as such uses are dependent upon students’ having access to relevant technology and internet at home, the latter sometimes due to resource but sometimes due to poor rural connectivity.

Some schools provide online tutoring for pupils who are behind in certain areas of the curriculum. Rather than do this during the school day, parents are asked to sign up to an alternative time when their child can access this support online from home. The advantage of this model is that the pupils do not miss out on other subjects although it can be challenging to find appropriate tutors when sessions are out-of-hours.

The use of mobile phones in lessons varies. Some schools ban their use in lessons whilst others encourage use such as when engaging in low stakes quizzes on platforms such as Kahoot. Students have access to the school Wi-Fi in this case. In one school extensive use is made of eBooks in lessons by staff and by sixth form students (though we understand that these are increasingly common elsewhere). This has led to a reduction in the number of physical textbooks available.

Common themes

There is some evidence from the interviews that the use of visualisers in lessons by teachers is increasing, examples including:

- A whole school project relating to metacognition in which teachers are encouraged to answer questions with think aloud approaches.
- Use of visualisers with manipulatives to aid understanding of mathematical structure.
- The visualiser is used as a tool for sharing pupils’ book work for discussion.

Interviewees teaching post-16 mathematics all report the use of graphing software when teaching functions and graphs; in most cases this included student use as well as teacher demonstration. Teachers believe that the ability to visualise and manipulate lead to deeper understanding. Whilst the use of graphing software is prevalent post-16, it was less commonly used in other phases. There was no reported use of technology for exploration of data across these ten schools.

There is evidence from this small number of interviews that the use of video is increasing. Commercially produced and teacher-made videos are used in a range of ways including:

- Where the teacher pre-records a short video of the use of a manipulative which an individual, or group of learners, can access independently.

- Where teachers use externally produced videos – such as those by White Rose Maths, or others found on You Tube – to provide an alternative voice or description in the lesson.
- Of students being directed to use videos as a means of 'catch-up' on areas of online homework which have not been completed successfully.

Professional Learning: Digital Technology

There is evidence from the interviews that professional learning is mainly undertaken by those staff who are interested in learning new skills to further their teaching repertoire. Furthermore, professional learning is reportedly undertaken voluntarily and in a piecemeal way. These points align with the findings from the survey, suggesting that systematic approaches to professional learning that encompassed all members of these schools/departments are uncommon.

Decisions about the equipment and software available to teachers are typically made by a school business manager. Although subject leaders have shared ideas for using the technology to support teaching and learning in their subject, there has not been a "pedagogy manager" taking overall responsibility for developing teachers' principled use of technology more generally.

Although these subject leaders noted the availability of training, the challenge was for teachers to find the time not only to attend the course but then to have the time to practise those skills to become confident enough to use it in lessons. Interviewees wanted more training on how to use pre-made resources in the classroom. Some training reportedly focused on creating your own resources, though concerns were also expressed about whether teachers have time to do this.

Summary

Bringing together the insights from the survey and from interviews a number of general conclusions can be drawn:

- The diversity of digital applications and tools in use is high, which results in patchy experience and expertise across educational settings and makes it challenging to ensure equitable access to associated professional support.
- Access to technology devices and resources is inconsistent and the reliability of equipment varies widely across settings.
- Some digital applications are perceived to be more prevalent and/or promising and these indicate possible starting points for a wider programme of professional learning.
- Practitioners are, on the whole, positive about the use of digital technologies in mathematics teaching and learning and would welcome a range of professional learning support.

Recommendations

> *Widespread access to phase-appropriate technology and to associated professional and technical support is needed to ensure system-wide equity for all practitioners and their learners.*
>
> *School leadership teams require support to develop appropriate digital strategies for mathematics. Such strategies need to consider the sustainable access to both the technology and the associated curriculum-design, pedagogic, and technical support for practitioners.*

There is a need for developing a cadre of digital technology 'champions' for mathematics who are a) knowledgeable about the digital technology landscape, b) can support sustainable change management in a rapidly evolving educational technology scene and c) can design, develop and deploy professional learning initiatives.

3. Digital technology in assessment and curriculum

This section addresses the impact of assessment on how technology is used in the teaching and learning of mathematics. We will make some recommendations for future changes to assessment that could have a positive impact on the use of technology.

Assessment has long been a driver of classroom practice. Whilst there is criticism of 'teaching to the test' - particularly when assessments do not adequately assess the desired range of mathematical skills and competences - it is understandable that many teachers, and students, direct their efforts to maximising learners' outcomes. The consequence of this is that a great deal of attention in classrooms is focussed on how to score highly in assessments. The use of technology is often absent or has low status in mathematics assessment, which has a knock-on effect on the use of those digital technologies in classrooms.

Although technology has low status in the assessment of mathematics across the UK it is possible to find statements in support of its use in the different nations' curriculum documents. Unfortunately, these statements are often presented as guidance and there are very few places where technology is required in examinations. Many teachers and students interpret the use of technology as a non-essential component of learning mathematics.

To fully realise the potential of technology for mathematics it is essential that it is integrated into the assessment at all levels. For this to be effective it will also require the curriculum to change. In this section we have made a choice of primarily focussing on the assessment and not the curriculum. This is based on a belief that previous changes to the curriculum that have tried to increase the use of technology have failed to realise this potential when technology was not integrated into the assessment.

This section identifies four areas where there is potential to increase the use of technology through changes to assessment. These cover different stages of education and reflect experiences from different countries within the UK:

- The use of calculating tools in primary mathematics.
- Embedding digital technologies in the online assessment of secondary mathematics.
- The use of technology for working with data in post-16 mathematics.
- The use of technology in university mathematics courses.

Using calculating and computational tools in primary classrooms

Prior to 2015/16, at the end of KS2, pupils in England completed three statutory paper-based mathematics assessments and were allowed to use a calculator in one of these. Because the use of calculators featured in the assessments, calculators were used in classrooms and it was common for a KS2 classroom to have a tray of devices. The National Numeracy Strategy produced a popular booklet of tasks for use with KS2 pupils. The tasks demanded the thoughtful use of a calculator to explore number patterns and structures, and many offered an opportunity for collaborative working.

In 2015/16, the use of calculators was removed from the statutory assessments and discouraged in the accompanying National Curriculum for England:

> *Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used.*

The consequence of this is that calculators are now rarely used in English primary schools. In an interview with a primary school mathematics leader, we were told that the primary "curriculum is busy enough" and teachers do not have time to include something that will not be assessed.

This has had two consequences. Firstly, because of the emphasis on securing written and mental arithmetic before learning to use a calculator, pupils are missing out on opportunities to develop their mathematical thinking and problem solving skills. Calculating tools enable pupils of all ages to focus on testing hypotheses and generalisations, and also enable them to access problems involving numbers that might be difficult or more time consuming for them to calculate using written or mental methods. Although counterintuitive to some, research indicates that, when used effectively, calculators are associated with improved mental and written calculation skills.[18]

Secondly, on entry to secondary school, pupils now lack familiarity with the calculating tools (including devices, apps and spreadsheets) that are used across mathematics, science and design & technology in KS3. In an interview with a mathematics adviser, we learned that Y7 pupils "delegate all responsibility to the [calculating] tool" because they have not been taught to use such tools as support for written and mental methods nor to estimate, approximate and interpret answers. The solution is not to oppose the idea of the calculator, but to be better at using such tools, and in the progression from physical devices (i.e. calculators) as the foundation for more advanced/virtual apps (e.g. on a smartphone).

The final – and perhaps most compelling – reason to incorporate digital technologies into mathematics lessons is that technology is an essential part of 21st century life. As well as calculating apps being available on every smart phone, pupils also meet both games and software outside of the classroom which enhance their understanding of numbers and shapes. If these tools were integrated into a high-quality primary mathematics curriculum, and associated assessments, pupil learning and their mathematical experiences would be richer.

Recommendation

> *Future reforms of primary statutory curricula (or non-statutory guidance) and assessment should explicitly address the use of digital technology in mathematics teaching across all primary years, and support for teachers to implement these changes should be provided. In particular, calculating tools such as physical calculators as well as digital calculator apps and spreadsheets should be (re-)introduced to enhance pupils' exploration and understanding of number, and to complement their learning of mental and written methods of calculation.*

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[18] Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). *Evidence for Review of Mathematics Teaching: Improving Mathematics in Key Stages Two and Three*. Education Endowment Foundation. https://educationendowmentfoundation.org.uk/public/files/Publications/Maths/EEF_Maths_Evidence_Review.pdf

Embedding digital technologies in the online assessment of mathematics

In the UK, most of the assessment of mathematics takes place using paper-based examinations. To conform to the examination rules, the technology that students are allowed will usually be restricted to a scientific or graphical calculator. In addition to this there are also some non-calculator papers. As a result, students are not tested on their *use* of mathematical technologies, beyond a few straightforward calculator functions such as recalling the values of trigonometric ratios, calculating roots of numbers and calculating using standard form notation.

Many of the specifications encourage the use of technology in the teaching and learning of the subject; however, as this is often not directly assessed it is easily overlooked by many teachers. This issue is further complicated by graphical calculators being allowed, but not mandated, for many examinations. This rule exists as it is perceived that requiring a graphing calculator would deny access to students who could not afford one. Consequently, a graphing calculator, although allowed, is not meant to give students an advantage in the examination. Many teachers infer from this that graphing technology is of little value when learning mathematics.

Free online graphing tools and graphing apps on mobile devices are now widely available. Unfortunately, as these tools are not valued, this results in the use of technology being limited and students benefit little from the technological innovations that have become readily available in recent years.

There is currently a move towards online examinations to replace paper-based ones[19]. There are issues around assessing mathematics in online environments, such as how to input and transcribe mathematics in a digital tool. It is beyond the scope of this report to address these issues. However, if online assessment were to be used it would open up the possibility of embedding digital tools such as graphing, geometry and spreadsheets into the testing platform.

If all students had access to embedded digital tools in the testing platform, and to the same tools in the classroom, then it would be feasible to assess students directly on their abilities in the use of technology for mathematics. To fully realise this potential the style of assessment questions and consequently the curriculum would need adapting. Furthermore, recent developments in speech and handwriting technologies are narrowing the gap between oral and written mathematics, which might enable opportunities to design and pilot more natural forms of digital assessment. More research is needed in this area.

Recommendation

> *As online assessment develops, the opportunity to embed mathematical digital technologies should be taken early. This should include the use of calculation tools, spreadsheets, graphing tools, and dynamic geometry software.*

The use of technology for working with data

The study and assessment of statistics has traditionally been based around using very small data sets. This contrasts with how technology is used for working with data in workplaces and in further/higher education. In practice, especially when working with large quantities of data, most of the processing

[19] See https://www.thetimes.co.uk/article/watchdog-looks-to-ditch-paper-a-level-and-gcse-exams-qn08s5r32

is outsourced to technology. Due to the current focus on the manual processing of data, we have a curriculum that mainly emphasises the largely outdated algorithmic approaches to calculating statistics and drawing charts. This limited approach does not prepare students well for working with data in advanced and higher study across the disciplines or in the workplace. Their skills in collecting data, validating data sources and interpreting results are underdeveloped. In recent years there have been attempts to address this issue at both A level and within Scottish Highers.

The changes to the A level Mathematics curriculum in England in 2017 included the introduction of the large data set. This was a data set that students were expected to study throughout their A level course.

Although the intention of this development was to encourage students to experience exploring data with technology, it has not had the widespread impact that was hoped for. One possible reason for this is that understanding of the associated curriculum knowledge and skills is assessed through conventional paper-based examinations where all students do not have access to relevant technology.

The design of the current assessments has not been perceived as rewarding students who have explored the data sets using technology. Many teachers perceive a pressure to prioritise teaching activities that are directly related to the assessment their students will undertake and consequently the large data set has been given little importance in lessons:

> *The introduction of the large dataset also generated significant uncertainty, in relation to the style of the questions as well as the extent to which students would need to be familiar with the dataset. A number of participating centres chose not to cover this element of the course or only to engage with it superficially, as they felt that the time and resources required to engage with it did not reflect the marks available; this trend increased over time*[20]

In Scotland, a new National Qualification - Higher Applications of Mathematics - was designed to broaden the pathways within Higher level qualifications to include areas such as statistics, mathematical modelling and finance. The first learners were presented for this in May 2022. The use of technology is embedded throughout the course, requiring learners to use spreadsheets and statistical software (such as R Studio or Minitab). The final grade is based on an exam and a statistics project in a ratio of 8:3. The use of technology is necessary in both assessment elements. At this level the emphasis moves away from algorithmic processes and focuses on the interpretation of statistical measures and calculations.

This new course has been designed in partnership with employers and higher education partners to help learners develop the skills they will need for their varied futures and has been welcomed by stakeholders as a positive step in embedding technology within learning and assessment.

Recommendation

> *The development of technological skills for working with data should be part of the curriculum for all students. The assessment of these skills is necessary for them to be embedded in the learning process. This could be by ongoing practical work or high-stakes exams where they have access to technology.*

[20] Redmond, B., Golding, J., & Grima, G. (2020). Teaching and learning for 'moving goal-posts': Reformed A Levels in mathematics. Proceedings of the British Society for Research into Learning Mathematics 40 (1).

Diverse technology use in university mathematics teaching

Technology is increasingly used in mathematics teaching and assessment across a variety of university disciplines[21], from STEM to economics and social sciences. According to some academics, already a decade ago, a typical mathematics degree consisted of 30-40% of modules that embedded technology, with some universities currently restructuring to increase this to 80%. A recent survey of higher education assessment of mathematics[22] noted an increase in skills, computational and programming modules overall but also an increase in the number and weight of final year projects using technology.

The reasons for this move reflect the nature of the subject, e.g. applied mathematics and statistics traditionally use software for modelling and working with data. Using technology in mathematics is also pragmatic, especially for automated assessment of large cohorts of students, and is essential for employability, reflecting the use of mathematics in modern workplaces. Additionally, technology is leading to a rethinking of teaching and assessment to benefit learners. It is worth noting that universities have relative freedom to design teaching and assessment so digital technology innovation is common.

Examples of using technology in mathematics include purpose-built software (e.g. Minitab), versatile programming tools widely used in workplaces (e.g. Python or R), numerical computational environments (e.g. MatLab) or unique tools developed for mathematicians (e.g. the editor tool LaTeX). Assessment of the modules where such technologies are used substantially diverts from traditionally hand-written exams and may include coursework, projects, presentations, automated quizzes, electronic workbooks and other electronic as well as hand-written outputs.

Automated assessment systems for mathematics have been available for almost two decades. STACK, NUMBAS, and DEWIS are some popular examples used in HE where a computer algebra system is integrated with an open-source learning management system, such as Moodle. Assessment systems typically allow teachers to generate multiple randomised versions of a mathematical task, check mathematical properties of students' answers and can provide automated feedback, which responds to students' answers in a way that is intended to improve their performance on the task[23].

Some elements of mathematics are more difficult to formalise such as proof and problem solving so a pragmatic approach is taken to assessment. While human-marking is preserved for assessing mathematical justifications and explanations which are submitted as traditional hand-written work, largely skill-based questions are assessed automatically. Nevertheless, research is constantly advancing, and auto-assessed questions are utilized in many areas of mathematics, including those associated with computation, coding, modelling, but also when constructing mathematical arguments. AI tools are expected to further impact mathematics assessment in HE. The Royal Statistical Society (RSS), London Mathematical Society (LMS) and Institute for Mathematics and its Applications (IMA) released a joint statement recently advocating for traditional invigilated tests as the best way to ensure security.

[21] A longer account of this section can be found in Lyakhova (2023) 'On the use of technology in university mathematics teaching and assessment in STEM degree schemes: discussion paper for JMC'.
[22] Iannone, P., & Simpson, A. (2022). How we assess mathematics degrees: the summative assessment diet a decade on. Teaching Mathematics and its Applications: an International Journal of the IMA, 41(1), 22-31.
[23] Sangwin, C. (2013). Computer aided assessment of mathematics. OUP, Oxford.

Experiences in higher education have implications for adopting technology in mathematics teaching and assessment more widely. Access to technological devices and corresponding equity issues should be considered. Some universities offer electronic devices on loan (similar to a book loan), while others provide alternative teaching and learning materials for students with limited access to technology.

Recommendation

> *The developments of digital technology in both university teaching and in a range of models for, and approaches to, assessment of mathematics and applied-mathematics programmes should be used to inform the innovation and design of pilot assessments for post-16 mathematics programmes.*

4. Digital technology and systemic change

Introduction

In the second half of 2022, the Working Group commissioned a rapid review to identify countries in which mathematics-specific digital technologies are widely and regularly used for learning mathematics at school (mainly secondary). The rapid review also aimed to explore how policy related to curriculum and assessment may have influenced what took place.

The identification of countries or states (education systems) to form the sample was informed by background reading and consultation with country experts. This included:

- Research reports (2015 onwards)
- International survey/assessment reports such as TIMSS
- Policy and curriculum documents
- Reports in the media

Following this, six country cases were selected, each offering relevant insights of how digital technologies have been taken up in mathematics education at scale:

- **Estonia**: From the 1990s, ICT in education has been a priority area. In mathematics a partnership with Wolfram Maths was established in 2013 to develop innovative teaching approaches in statistics.
- **Ireland**: There is a digital strategy for schools and recent curriculum reforms (Project Maths) have encouraged the use of GeoGebra at secondary level.
- **Germany**: Many education policies are devolved to the 16 states. There is a wide variety of practice but at the federal level a 'digital pact' aims to provide digital infrastructure in schools. It is possible that there are some successful innovations at state level.
- **Denmark**: The use of Computer Algebra Systems (CAS) is mandatory at upper secondary level, and assessment requires the use of CAS. There is also interest in computational thinking (CT) here and across the Nordic nations.
- **Australia** (Victoria): Education decisions are devolved to each state. In senior secondary mathematics there is an explicit expectation that technology will be used and examinations require the use of CAS.
- **Norway**: It is a requirement to use digital technologies across primary and secondary education, and digital technologies appear to be widely used in both phases of education. Like Denmark, there is an interest in CT.

Country vignettes

The themes that came out of the grey literature related to curriculum reform includes: 21st century skills; access to digital technology (e.g. the internet) and numbers of students per computer; equity in terms of access to technology; teacher professional development; and frameworks for teacher 'digital competencies'. It appears that most (all) curricula encourage the use of digital technologies in mathematics teaching but few mandate such use in teaching or in assessments, with the exception of calculators. Outline summaries of findings from the case study countries are set out below.

Estonia

Estonia is a small country (1.3 million) which has established a strong programme of e-governance. Education is centrally controlled, and all schools follow a national curriculum. For the school-leaving examinations, students are examined in three subjects: Estonian language (or Estonian as a second language), mathematics and one additional subject chosen by the student.

Information and communications technology (ICT) in education has been a priority area for Estonia from mid-90s. In 1996, the Tiger Leap Program was launched to support the development of ICT infrastructure in schools, provide basic ICT training for teachers and develop educational software and learning resources in Estonia. In order to coordinate these developments in general education, the Tiger Leap Foundation was established in 1997. In 2012 the ProgeTiger programme, which built on Tiger Leap by providing schools with instructional resources and teacher professional learning to develop students' digital literacy skills, was launched. In 2016, the Ministry of Education and Research launched an online resource library called e-Koolikott ("e-Schoolbag") with the goal of making the full range of primary and secondary education resources available in digital form by 2020. (Aru-Chabilan, 2020; Põldoja, 2020).

In mathematics, between 2013 and 2016, the Estonian Ministry of Education and Research funded a project on Computer Based Statistics (CBS), developed by Wolfram Maths, which aimed at a fundamental change for learning data and statistics at lower and upper secondary levels. It was piloted but further funding was not forthcoming. The project was measured against progress on the previous assessment tasks, many of which were incompatible with a computer-based approach.

There is no evidence that the CBS approach has been widely adopted or that it continues. The country expert who was consulted about the use of ICT in subject teaching reported that it was not widely used and that it remained 'gimmicky'.

In-country expert: Artur Taevere

Ireland

Primary education lasts for eight years with post-primary education following for a further five to six years. It is divided into a 3-4-year junior secondary cycle, followed by a 2-year senior secondary school cycle leading to the award of the Leaving Certificate.

Project Maths (introduced between 2010 and 2012) was an ambitious reform of the Irish post-primary mathematics curriculum and involved changes to what students learnt, how they learnt it and how they were assessed. It reshaped the curriculum in line with a philosophy of mathematics education that highlighted solving problems, especially those set in real-life contexts.

Early evaluations of Project Maths provided evidence of the positive impact on students' attitudes towards mathematics and their achievement at an individual strand level (Jeffes et al. 2013). However, there were indications of problematic areas. The research identified differences between the intended and implemented curriculum; teachers were still "not really 100% sure what to do".

Project Maths encouraged the use of digital technologies, mostly Geogebra, but ICT/technology is hardly mentioned in the reports of the project. The country expert suggests that use of mathematics-specific ICT is not regular or widespread and that teachers are not convinced of the value of using it because they are 'constrained' by the terminal exam.

In-country expert: Aibhín Bray

Germany

The schooling system varies throughout Germany because each state (Land) decides its own educational policies. However, in nearly all states young children first attend Grundschule (primary or elementary school) for 4 years from the age of 6 to 9. Then they have to decide which track they want to follow. The main tracks are Gymnasium, Realschule and Mittel- or Hauptschule.

Germany's secondary education is separated into two parts, lower and upper. Lower-secondary education in Germany is meant to teach individuals basic general education and gets them ready to enter upper-secondary education. In the upper secondary level Germany has a vast variety of vocational programs. German secondary education includes many different types of school.

Germany has had poor access to computers and tablets for students. Concerning state-supported programmes such as "Gute Schule 2020" (good school 2020) or the "Digitalpakt" (digital pact; €5 Bn) which are designed to drive the digital revolution, the situation changed in recent years.

It is difficult to find any reports of widespread, regular use of any technologies in the teaching and learning of mathematics.

In-country expert: Hans-Georg Weigand

Denmark

Danish education is highly decentralised and runs from early childhood education and care (ECEC) to upper secondary. The *folkeskole* (people's school) covers the entire period of compulsory education, from the age of 6 to 16, encompassing pre-school, primary and lower secondary education. School autonomy levels are high and municipalities have extensive responsibilities in primary and lower-secondary schooling. Policymaking therefore depends heavily on the ability of different actors to collaborate and co-ordinate effectively.

In 2005, computer algebra systems (CAS) were introduced into the Danish upper secondary school mathematics program as a mandatory component of mathematics teaching and learning. Ministerial orders required that CAS be used not only to assist with symbolic manipulations, calculations and problem solving, but also to support the development of skills and assist mathematical concept formation. CAS is also an integral part of the national written assessments.

Research reports suggest that, while the use of CAS may have been widely accepted, its use may also have had some unintended consequences. It is used to solve equations, differentiate expressions and so on, but less for exploring concepts.

The country expert suggests that, at primary level, GeoGebra is used but there is no evidence that it is used widely and regularly.

In-country expert: Uffe Thomas Jankvist

Victoria, Australia

In Australia, education is decentralised, with each state setting its own educational policies. However, all schools are expected to follow a national curriculum. Education is compulsory between the ages of about 5 and 16, with slight variations depending on the state and the birthday of the child. Each state is responsible for issuing certificates and/or qualifications to secondary students, collectively referred to as the Senior Secondary Certificate of Education. Students in Years 3, 5, 7 and 9 participate in the National Assessment Programme – Literacy and Numeracy (NAPLAN) taking a series of standardised tests.

In the state of Victoria, the senior secondary school curriculum and its assessment is specified and controlled by the state. In mathematics, a stronger endorsement and expectation of technology use in Victoria is one of the main differences from other Australian states and territories. The focus of regular and widespread use of technology has been mainly at upper secondary level (e.g. the use of CAS). The final mathematics examinations involve school-based and external assessments, both of which require the use of CAS. However, students are also required to sit a 'no technology' examination, included to address public confidence that students are learning important aspects of mathematics.

The introduction and integration of digital technologies was carefully designed and managed by a team involving the systems mathematics manager, a researcher and leading teachers, within a relatively small context. According to the country expert "Everyone uses CAS if they do Year 12 maths exams."

In-country expert: Kaye Stacey

Norway

Norway is a relatively small country (population c. 5 million) and has a decentralised education system. Technology has been an element of the Norwegian general education curriculum since 1997. In 2006, curriculum reform specified "Use of digital tools" as one of five "basic skills" as a binding requirement for all grades (1-12: age 6-7 to 17-18). However, this had limited impact on the use of technology in schools.

In 2015, a new assessment system was introduced in which pupils were required to use digital tools. In mathematics, this meant that pupils had to use digital tools to solve some of the test problems. As a result, the use of digital tools increased; pupils had more experience of GeoGebra and spreadsheets in the lower grades, and of GeoGebra and Computer Algebra Systems in the higher grades.

In 2020, further reform revised the *digital tools* basic skills to "Digital competence" and introduced computational thinking into the curriculum across all subjects.[24] Between 2017 and 2021, substantial

[24] Bocconi, S., Chioccariello, A., & Earp, J. (2018). *The Nordic approach to introducing Computational Thinking and programming in compulsory education. Report prepared for the Nordic@BETT2018 Steering Group*. Nordic@BETT2018 Steering Group. https://doi.org/https://doi.org/10.17471/54007

funding was provided by the Norwegian government for schools to purchase digital teaching aids. Considerable policy attention is directed at this strand of education. For example, Norwegian Agency for Education and Research collects data on education and this is regularly reported in The digital state in schools and kindergartens, which is one of six topics in an annual data-informed report on the state of education in Norway.[25] In 2021, an expert group was established to provide recommendations and guidance on professional development for teachers.

As a result of these initiatives, the infrastructure for technology in schools is very good and the use of digital technology by pupils is very high. Most pupils have access to their own device at school and at home, and, moreover, the digital disadvantage divide is relatively narrow.

Despite these successes, the system faces challenges. There is no data on how teachers use digital tools for teaching, nor about how much the teachers use computational thinking in their teaching. Teachers are considered to lack sufficient digital competence to teach computational thinking effectively.

The country expert commented: "These [good infrastructure and data on the use of digital technologies] are necessary conditions for using technology in the classroom. But they are not enough. The teachers need to be trained in how to use technology in their teaching and they need to believe that technology is a good tool for teaching and important for the pupils. With the change in examination, the teachers have started to use technology more, but they still need more training."

In-country expert: Tor Espen Kristensen

Summary

- Some regions/systems appear to have achieved widespread, frequent and regular use of digital technologies in mathematics classrooms.
- National or regional initiatives tend to focus on provision of hardware and Internet access, and often professional development for teachers. These initiatives are not always formally evaluated. The Norwegian case demonstrates the value of evaluation and continuing data collection on implementation.
- It is increasingly recognised that it is necessary for schools to have well-maintained and up-to-date hardware and software. There is a substantial cost to providing and maintaining this infrastructure. However, the infrastructure alone is not sufficient to ensure that digital technologies are embedded in teaching and learning.
- In addition, there need to be incentives for schools and teachers to integrate digital technologies into teaching and learning. Requiring, or encouraging, digital technologies use in high-stakes assessments appears to be particularly important, but schools and teachers also need access to professional development opportunities.
- The well-publicised strategies adopted by Estonia were/are more related to the provision of resources than subject-specific teaching and learning strategies or approaches. The Computer Based Statistics project seems to have stalled.

[25] https://www.udir.no/tall-og-forskning/publikasjoner/utdanningsspeilet/utdanningsspeilet-2022/

- Widespread adoption of digital technologies was only found at upper secondary level and involved CAS and the use of CAS in national examinations. It generally seems to involve the technology in 'doing the mathematics' rather than exploring concepts. There is little evidence of the use of other digital technologies.
- It appears to be easier for systems to adopt digital technologies for mathematics teaching and learning at upper secondary level. Aside from Norway, there is limited evidence of widespread digital technology use elsewhere.
- It is important to align all aspects of the teaching and learning of mathematics, including curriculum, textbooks, assessment and support for teachers, and to maintain high expectations that technology will be used. Policymakers, school leaders, teachers, parents and students need to recognise the value of using the digital technologies (e.g. by requiring their use in examinations).
- Adoption of digital technologies requires careful planning and a continuing focus on implementation.
- Use of digital technologies in mathematics can lead to students engaging with mathematics in unexpected ways, some of which may not align with the values of teachers.

Recommendations

From across these country cases the following tentative recommendations can be made, though these need to be synthesised with those from elsewhere in the Report in Section 5:

> *There is a need for system-level recognition that assessment drives what happens in classrooms. If, therefore, decision makers want to improve the use of digital technologies in mathematics classrooms, examinations will need to require their use so that schools and teachers will be motivated to use them.*
>
> *In any large-scale change programme, it is important to plan for, and build in, high-quality formative evaluation from the outset. This applies to both system-wide improvement planning and to qualification changes at the sub-system scale.*
>
> *Professional development programmes for teachers should focus on the digital technologies that teachers (and students) actually use, i.e. an assets-based approach, and considerations of how to make the best use of them.*

5. Concluding recommendations

Digital technology and mathematics education

The motivation for the present Report, and for the JMC Working Group's activity that preceded it, was an acute sense that despite the promises of digital technology to enhance mathematics education, and the ongoing transformation of all aspects of modern society by technology, little has changed since the publication of the 2011 JMC report and very little progress had been made against that report's recommendations.

Perhaps it would be more accurate to say that in practice a huge amount has changed in the application of technologies in all aspects of the education system, but that when it comes to mathematical education this has been highly organic, patchy, inequitable and with little evidence of widespread impact. This has been due, in no small part, to increasingly complex educational systems, political focus on wider reforms of curriculum and qualifications, and of course to the disruption of a global pandemic at the turn of the decade.

That said, in HEIs, there has been a rapid shift in the use of digital technologies and a genuine effort to use the kinds of tools that professional mathematicians and users of mathematics will engage with in their employment futures. This leaves a considerable gap between the use of digital technologies in schools and the kinds of uses that are made of these in other subjects (e.g., KS3 Science in England) and in the jump from compulsory education to higher education and work. In Scotland, the innovative Application of Mathematics Higher is demonstrating that change is possible and that assessments really must value the kinds of technology used in enacted curricula in classrooms.

The challenges facing the UK nations in adopting digital technologies effectively into the teaching, learning and assessment of mathematics are not unique. The country vignettes highlight the considerable barriers to achieving such systemic change, but that change is possible under the right conditions. Unfortunately, our collective understanding of implementation science, including the conditions under which sustained and effective change strategies in complex education systems can be designed for, implemented, and evaluated, is limited.

The careful reader will have observed a number of recurring themes and resonances in the different aspects of the Working Group's activity (e.g., assessment, equity of access). The report therefore concludes by synthesising the recommendations from the above sections. In addition, we try to hold to the principle of setting out a) realistic, small steps which, if done well and widespread, would amount to major change, and b) achievable actions, albeit ones that will require the coordinated actions of a range of stakeholders.

Key recommendations

Recommendations on curriculum reform

1. **Future reforms of statutory curricula** or non-statutory guidance should explicitly address the use of digital technology in the teaching of mathematics and data analysis across all years. Well-designed support for the implementation of reforms should also be provided.
2. **Computational tools should be actively encouraged** (e.g., digital and physical calculators, spreadsheets) in primary and secondary classrooms to enhance pupils' exploration and understanding of number, and to complement their fluency with mental and written methods of calculation.
3. **Universities' use of digital technologies** in mathematics and applied-mathematics courses should inform the innovation and design of pilot post-16 mathematics programmes.

Recommendations on assessment

4. **Appropriate high-stakes assessment of digital technology skills** is needed to ensure that such skills are embedded in, and developed throughout, the learning process.
5. As online assessment develops, the **opportunity to embed mathematical digital technologies must be taken early**. This should include the use of computational and graphing tools, and dynamic geometry tools as appropriate. This will require significant development work by awarding organisations and regulators.
6. **National bodies should ensure that skills for working with data are better assessed**, through ongoing practical work or high-stakes examinations in which students have access to appropriate digital technologies.

Recommendations on professional development

7. All **professional development for teachers of mathematics should embed appropriate uses of digital technologies** such that they become normalised, rather in the same way that digital technologies should be embedded in all teaching and learning.
8. **Professional development programmes on digital technology** should focus on the effective use of digital technologies that teachers (and learners) use frequently in order to increase the likelihood of widespread adoption.
9. **A cadre of digital technology 'champions' for mathematics** should be developed, who a) are knowledgeable about digital technologies for mathematics education, b) can support sustainable change management in a rapidly evolving educational technology landscape, and c) can design and deliver professional high-quality initiatives for teachers of mathematics.

Recommendations on implementation, resourcing and leadership

10. **Common access to a *limited* and *agreed* set of phase-appropriate digital technologies** and associated professional and technical support is needed to ensure system-wide equity and coherence for all practitioners and their learners.
11. **Support for leadership teams to develop appropriate digital strategies** for mathematics is needed. This must ensure sustainable access to both the technology and the associated technical, pedagogic, and curricular (institution-based) support for practitioners.

12. **High-quality formative evaluation** of any large-scale intervention using digital technologies for mathematics education is essential and should be included in the implementation design from the outset.

Next steps

The promises of digital technologies for enhancing many varied aspects of mathematics teaching and learning, from reception to undergraduate study, have yet to be fully realised. Indeed, given the rapid pace of technological change and the slow lead time for designing and implementing systemic change in the education system, this is likely to continue to be the case.

We hope that the findings from the Working Group's activities and the above recommendations can further contribute to the debates, change leadership and policymaking activity in mathematics education and related areas. Furthermore, we are of the view that many of these changes could have considerable impact, particularly if implemented in a well-orchestrated way.

Arguments about the value of mathematics are well-made and widely accepted. It is now time to ensure that the digital tools that are now commonplace, both for doing mathematics and for applying mathematical and data ideas in a range of education, work, and life contexts, are widely and equitably adopted to enhance mathematical attainment and outcomes for all.

6. Appendix

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