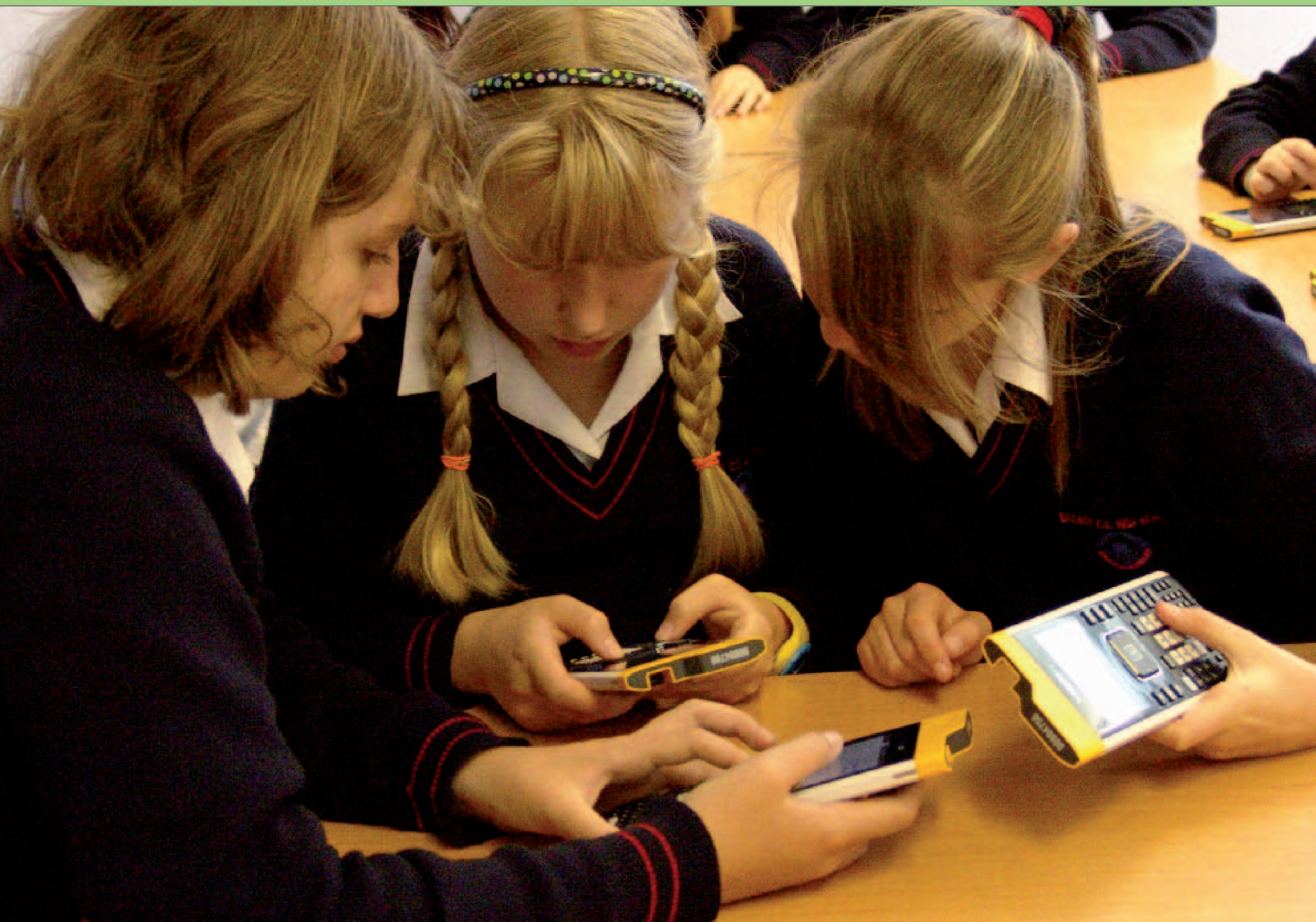
JMC

Joint Mathematical Council
of the United Kingdom

Digital technologies and mathematics education

A report from a working group of the
Joint Mathematical Council of the United Kingdom
Chaired by Professor Rosamund Sutherland.
Edited by Dr Alison Clark-Wilson, Professor Adrian
Oldknow and Professor Rosamund Sutherland
September 2011



Contents

Digital technologies and mathematics education

Executive summary and recommendations

1. Scenario

1.1 In order to provide a context for the body of the report we invite readers to imagine they are travelling on a train together with students on their way to school. One student is finishing off his mathematics homework and struggling to solve some quadratic equations by factorisation. Another student, who has already diligently done her homework, is playing a realistic action video game on her smart phone. This scenario is, of course, designed to emphasise the stark contrast between the worlds of current (and past) mathematics education at school and the world in which many of our current students live most of their life.

The point about modern smart phones and other portable digital technologies is that they are not just phones. They are multi-purpose computers with built-in processors, memory, colour display, audio playback, wireless telephone and broadband communications, Global Positioning Systems and accelerometer sensors, still and video camera, touch screen input – and they also run a wide variety of Apps. These ‘Apps’, short for Applications, are what used to be called computer programs or software. The relevance of this choice of analogy is that the girl could quite as easily have been using an Internet browser to access mathematical information, or discussing her maths homework by phone with a friend, or using Google Maps and Google Earth to plan a cycle trip or using a powerful, free, mathematical tool such as GeoGebra to explore an interesting mathematical problem.

We can also look more deeply at some of the sub-components listed above. The microprocessor, which powers the first of the smartphones, and most of the current portable computing devices, was designed by a UK company and is a direct descendent of the knowledge and skills which went into developing the Acorn Archimedes computer in Cambridge. The aesthetic design and the engineering specification for this, and many other Apple products, come from a team led by a British born and educated designer. The encryption which makes secure mobile communications possible was developed by a British mathematician. The Internet is based upon the designs of a British computer scientist. Video games are a major source of income for the UK economy and require advanced skills of mathematics, physics, biomechanics and computer programming to achieve virtual reality. Global Positioning Systems and the Internet rely on communications satellites – a technology in which the UK is a world leader.

What is needed in schools and colleges is student-led mathematical modelling, problem solving and computer programming which makes use of the powerful mathematical digital technologies that are widely used in society and the workplace.

2. Introduction

Einstein famously said that his pencil was more intelligent than he was - meaning, that he could achieve far more using his pencil as an aid to thinking than he could unaided. There is a need to recognise that mathematical digital technologies are the ‘pencils of today’ and that we will only fully exploit the benefits of digital technologies in teaching, learning and doing mathematics when it becomes unthinkable for a student to solve a complex mathematical problem without ready access to digital technological tools.

2.1 The Joint Mathematical Council of the UK was established in 1963 to promote the advancement of mathematics and the improvement of the teaching of mathematics. It brings together representatives from societies and organisations which are involved with the processes and products of mathematics education. Its meetings are attended by academics and professionals concerned with pure mathematics, applied mathematics, statistics, operational research, higher education, research and commercial applications of mathematics as well as representatives from government departments and agencies together with teachers, advisers, inspectors and teacher educators.

2.2 Against a background of widespread concern about the UK’s ability to meet the increasingly technological skills needs of major sectors of the economy, the JMC established a working group to consider the role which digital technologies might and should have in mathematics education, now and in the near future.

3 What are digital technologies?

3.1 Digital Technologies refer to a wide range of devices which combine the traditional elements of hardware (processing, memory, input, display, communication, peripherals) and software (operating system and application programs) to perform a wide range of tasks. They include: technical applications; communication applications; consumer applications and educational applications.

3.2 The generic term `computing’ has been replaced with the term `Information Technology’ (IT) in most walks of life. The National Curriculum introduced the term `Information and Communications Technology’ (ICT) in the educational context – which became one of the core subjects. ICT in schools has concentrated on the use of generic software such as word-processors, spreadsheets and presentational tools together with tools for digital communication such as e-mail and the Internet. In order to avoid confusion with this interpretation of ICT the term ‘digital technologies’ is used throughout this report.

4. Why are digital technologies important to the economy and what contribution can education make?

4.1 A key issue facing the UK is how to inspire and develop the next generation of innovators, creators, scientists, technologists, engineers and mathematicians on which our future well-being and economy depends.

4.2 Policy makers at the European level state that increasing young people’s interest in Mathematics, Science and Technology is essential for sustainable economic growth in Europe.

4.3 When comparing the UK to China and India Michael Gove said "It's rocket science – mathematics, engineering, physics and the other hard sciences which are driving innovation globally, and generating growth for the future" (Gove, M. The answer is rocket science. 2008; Available from: http://conservativehome.blogs.com/torydiary/files/gove_on_science.pdf).

4.4 There is a need to build on and improve the UK's capacity for technological innovation and creativity. Education at all levels has its part to play in engaging the interests and enthusiasm of young people so that they pursue education, training and career paths which contribute to the nation's needs while themselves achieving satisfaction and reward.

5. How are digital technologies impacting on mathematical practices outside school and college?

5.1 Mathematicians work on the important practical issues of their era, which have always required both the development of the subject together with the tools to support this. Pascal's and Leibniz's mechanical calculating machines, Napier's logarithms, Babbage's difference engine, Newman's Colossus and Turing's Bombe for crypto-analysis at Bletchley Park are just a few examples of computational tools which have been fundamental to the evolution of digital technologies to support mathematical developments. In this respect mathematics is, and has always been, a dynamic problem solving activity for which humans have continued to develop and exploit new tools.

5.2 New kinds of jobs are now open to people with mathematical qualifications, especially those with skills in the use of digital technologies. It is often argued that the number of mathematical tool users will outweigh many times the number of tool producers, with the implication that only a few experts need to really understand how to produce a mathematical tool. Such dichotomies are unhelpful as the workforce of the future will need to include a large number of mathematical tool modifiers. For the tool modifier, the central issue becomes the articulation of what the digital technology should do, designing a system to do it, and verifying that it will perform as specified.

5.3 The vast majority of young people are involved in creative production with digital technologies in their everyday lives, from uploading and editing photos to building and maintaining websites. They acquire many skills which will be relevant in their careers, but which are not drawn on during their time in school. They acquire new skills rapidly, and share their knowledge with their peers – but rarely in an educational context.

5.4 Whereas digital technologies are the 'tools of the trade' of the modern scientists, technologists, engineers and mathematicians, young people are not likely to use digital technologies for the creative production of science, technology and mathematics in their everyday lives.

Recommendation 1: For policy makers and teachers

School and college mathematics should acknowledge the significant use of digital technologies for expressive and analytic purposes both in mathematical practice outside the school and college and in the everyday lives of young people.

6. How are digital technologies impacting on mathematics education in the UK currently?

6.1 Ofsted reports have concluded that technology is underused within mathematics and that its potential is generally underexploited. Usage of digital technology within school mathematics has been predominantly teacher-led and mainly focused on presentational software such as PowerPoint and interactive whiteboard software.

6.2 Unless we can develop mathematics education in a more stimulating way, which takes into account the modern world and students' interests we are in danger of turning mathematics into an increasingly 'dead language' and alienating groups of students whose mathematical potential will remain undeveloped.

6.3 Rote learning of the current mathematics curriculum will not be sufficient to produce the problem solvers, independent thinkers and designers that the country needs.

6.4 The barriers to a more creative student-focussed use of digital technologies include:

- a perception that digital technologies are an add-on to doing and learning mathematics;
- current assessment practices, which do not allow the use of digital technologies, particularly within high stakes assessments;
- inadequate guidance concerning the use of technological tools within both statutory and non-statutory curriculum documentation.

6.5 Learning in science, technology and engineering in schools and colleges could be greatly enhanced if students were able to use digital technologies to perform mathematical processes, mirroring the types of applications used in STEM-based applications in the workplace. The benefit of using digital technologies relates both to the processing power afforded by the technologies and the opportunities to access real-world data, which is engaging for students.

6.6 There is already a wide range of existing 'mathematical' digital technologies which could readily be used by schools and colleges such as:

- Dynamic graphing tools
- Dynamic geometry tools
- Algorithmic programming languages
- Spreadsheets
- Data handling software and dynamic statistical tools
- Computer algebra systems
- Data loggers, such as motion detectors and GPS
- Simulation software.

6.7 With respect to the content of the new mathematics curriculum, it should include:

- the specification of the knowledge and skills required to use digital technologies within mathematical modelling and problem solving activities across a range of subject areas;

- student-led mathematical modelling and problem solving, which make use of the powerful mathematical digital technologies that are widely used in society and the workplace;
- a component of computer programming, interpreted in the widest sense of creating and communicating a set of instructions to a computer for a clear purpose.

6.8 Change to the mathematics curriculum and its assessment, although necessary, will not be sufficient to develop the classroom practices of teachers, many of whom have limited professional knowledge and experience of the type of digital technological tools being described in this report.

6.9 There will be a continuing need to update the skills of the teaching workforce, something which the subject associations and teacher education institutions working alongside industry and schools are well-placed to achieve.

Recommendation 2: For policy makers

Curriculum and assessment in school mathematics should explicitly require that all young people become proficient in using digital technologies for mathematical purposes.

Recommendation 3: For policy makers

High-stakes assessment needs to change in order to encourage the creative use of digital technologies in mathematics classes in schools and colleges.

Recommendation 4: For policy makers and school leaders

As the development of a technologically enriched student learning experience occurs at the level of the classroom, such change has to be supported by school leaders and accompanied by sustained professional development opportunities for teachers.

Recommendation 5: For policy makers

The UK Departments for Education and for Business, Innovation and Skills should establish a Task Force to take the lead in bringing together various parties with appropriate expertise to take forward the recommendations of this report and advise the Departments on required policy initiatives.

Digital technologies and mathematics education

Main report

1. Scenario

In order to provide a context for the body of the report we invite readers to imagine they are travelling on a train together with students on their way to school. One pupil is finishing off his mathematics homework and struggling to solve some quadratic equations by factorisation. Another pupil, who has already diligently done her homework, is playing a realistic action video game on her smart phone. This scenario is, of course, designed to emphasise the stark contrast between the worlds of current (and past) mathematics education at school and the world in which many of our current students live most of their life. The point about modern smart phones and other portable digital technologies is that they are not just phones. They are multi-purpose computers with built-in processors, memory, colour display, audio playback, wireless telephone and broadband communications, GPS and accelerometer sensors, still and video camera, touch screen input – and they also run a wide variety of Apps. These “Apps”, short for Applications, are what used to be called computer programs or software. The relevance of this choice of analogy is that girl could quite as easily have been using an Internet browser to access mathematical information, or discussing her maths homework by phone with a friend, or using *Google Maps* and *Google Earth* to plan a cycle trip or using a powerful, free, mathematical tool such as *Geogebra* to explore an interesting mathematical problem. We can also look more deeply at some of the sub-components listed above. The microprocessor, which powers the first of the smartphones, and most of the current portable computing devices, was designed by a UK company and is a direct descendent of the knowledge and skills which went into developing the Acorn Archimedes computer in Cambridge. The aesthetic design and the engineering specification for this, and many other Apple products, come from a team led by a British born and educated designer. The encryption which makes secure mobile communications possible was developed by a British mathematician. The Internet is based upon the designs of a British computer scientist. Video games are a major source of income for the UK economy and require advanced skills of mathematics, physics, biomechanics and computer programming to achieve virtual reality. GPS and the Internet rely on communications satellites – a technology in which the UK is a world leader.

2. Introduction

2.1 The Joint Mathematical Council was established in 1963 to promote the advancement of mathematics and the improvement of the teaching of mathematics. It brings together representatives from societies and organisations which are involved with the processes and products of mathematics education. Its meetings are attended by academics and professionals concerned with pure mathematics, applied mathematics, statistics, operational research, higher education, research and commercial applications of mathematics as well as representatives from government departments and agencies together with teachers, advisers, inspectors and teacher educators. It has produced major reports at the request of the Royal Society on the teaching and learning both of Algebra and Geometry.

2.2 Against a background of widespread concern about the UK's ability to meet the increasingly technological skills needs of major sectors of the economy, JMC established a working group to consider the role which digital technologies might and should have in mathematics education now and in the near future.

3. What are digital technologies?

Digital technologies (DT for short) are the modern developments of the so-called micro-chip which began to permeate computing applications in the late 1970s. For the purposes of this report we have chosen to concentrate on four different, though connected, sets of applications: technical, communications, consumer and educational.

Technical applications

These include the tools used by designers, researchers and engineers in the process of building new products and devices, solving new problems, carrying out research, monitoring systems, taking measurements etc. They include geometrical tools like CADCAM, computer algebra systems with symbolic manipulation, probabilistic simulation tools, tools for statistical analysis and forecasting, data-logging and modelling software etc. We can consider these as the digital "tools of the trade" of modern scientists, engineers, technologists and technicians. They also include sophisticated electronic aids such as industrial robots, unmanned weapons and medical imagery.

Communication applications

These include the tools used in many walks of life to collaborate at a distance with colleagues, such as e-mail, mobile-phone text messaging, social-networking, video-conferencing and document-sharing. With high speed broadband connections we are seeing a shift from people working alone with software installed on stand-alone computers towards remote interactions, possibly collaborative ones, using Internet-based software – so-called "Cloud computing". Young people in particular use a variety of websites and tools for their social interactions such as arranging their social life and keeping in contact with friends and family, keeping public diaries (blogs and *BEBO*), expressing opinions (*Twitter*), sharing photographs (*Flickr*) and posting videos (*YouTube*). Communities set up shared areas for the exchange of information (*Wikis*).

Consumer applications

These include a huge variety of ways in which things affecting our daily lives now rely on digital technologies. Consumer products such as cookers, vacuum cleaners and freezers have built-in control and sensor systems. Cars have sophisticated diagnostic and management systems as well as driving aids such as Sat-Nav, reversing imagery and even automated parking. Home and mobile entertainment devices such as TVs, players, pods, smart phones and pads provide music, radio, television, satellite broadcasting, 3D films etc. with ever increasing clarity and sophistication. Digital cameras now offer great clarity as well as high-speed and high-definition video modes. New generations of computer video games, such as the *Nintendo Wii!* and *Microsoft Kinect*, make use of sophisticated wireless sensor systems and stereo video imagery to predict displacements.

Educational applications

The UK has been the major adopter of an educational technology known as the Interactive Whiteboard (IWB) which enables teachers and students to interact with images displayed from a computer via a fixed digital video projector displayed onto a touch sensitive display surface. More portable methods of supporting whole-class interactions are based around laptop computers and lightweight mobile video-projectors. Nearly all schools and colleges now have their own websites through which teachers, students and parents can access a wide variety of information as well as exchange information and views. “Learning platforms” provide a means whereby users can access a variety of learning resources such as worksheets, homework tasks, supporting information, links to useful sources as information. There is also a range of portable devices for data-capture as well as digital feedback and control.

Summary

3.1 Digital technologies refer to a wide range of devices which combine the traditional elements of hardware (processing, memory, input, display, peripherals) and software (operating system and application programs) to perform any one of a given range of tasks. They include technical applications, communications applications; consumer applications and educational applications.

3.2 The generic term `computing’ has been replaced with the term `Information Technology’ (IT) in most walks of life. The National Curriculum introduced the term `Information and Communications Technology’ (ICT) in the educational context – which became one of the core subjects. ICT in schools has concentrated on the use of generic software such as word-processors, spreadsheets and presentational tools together with tools for digital communication such as e-mail and the Internet. In order to avoid confusion with this interpretation of ICT, the term ‘digital technologies’ is used throughout this report.

4. Why are digital technologies important to the economy and what contribution can education make?

The UK, along with most other Western nations, is looking to recover from the recent financial crisis through increased growth and improved international competitiveness of important sectors of the economy. There will always be a need for large scale engineering projects, such as in high-speed railways and more efficient generation of electricity. There is also a need to design and apply new technologies such as genetic-engineering, nano-technology and smart materials as well as developing ever better medicines, faster communications and more efficient vehicles. The UK is also a world leader in the lucrative field of digital entertainment through special effects and animations in the media of film, television and video games, including 3D. A key issue facing the UK is how to inspire and develop the next generation of innovators, creators, scientists, technologists, engineers and mathematicians on which our future well-being and economy depends.

Reports from employers such as the CBI, CIHE, Engineering UK and sector skills' councils show an existing and widespread skills crisis in many important sectors of the UK economy. In some areas, such as aerospace, there is a compounded problem with a substantial percentage of the skilled workforce nearing retirement. The Royal Academy of Engineering's *ICT for the UK's Future* report [1] discusses the implications of the changing nature of Information and Communications Technology and highlights the vital need for skills in specialised aspects of technical computing. The *Ingenious Britain* report [2] by Sir James Dyson sets out a strategy for industrial growth, innovation and international competitiveness which the government is currently implementing. Digital technologies are both fundamental to the way businesses work, and are themselves a major source of income for the UK's 'knowledge economy'. The twin drivers of skills and growth require a more technically educated workforce at all levels – from the creative innovators designing new products and technologies, right through to a new level of skilled technicians.

The Dyson report places education firmly at the centre of meeting our future skills needs. The Science And Learning Expert Group of the government's Department for Business, Innovation and Skills *Science and Mathematics Secondary Education for the 21st Century* report [3] makes concrete recommendations about what changes are needed in schools to support this. The recent *Next Gen.* report [4] from Ian Livingstone and Alex Hope for Ed Vaizey, the Minister for Culture, Communications and the Creative Industries Department of Culture, Media and Sport similarly identifies the key contributions schools can make in nurturing the computing, mathematical and artistic skills required by the digital media. The concern about how schools can help foster the computing skills currently in such short supply is at the heart of the current Royal Society Enquiry on *Computing in schools and its importance and implications for the economic and scientific wellbeing of the UK*.

Learning in science, technology and engineering is underpinned by an understanding of mathematics. In particular the study of science relies on the collection, analysis and interpretation of data.

Within Europe the lead has been taken by a group of around 45 chief executives and chairmen of major multinational companies of European parentage covering a wide range of industrial and technological sectors known as the European Round Table (ERT). In its own words:

> ERT is an informal forum bringing together around 45 chief executives and chairmen of major multinational companies of European parentage covering a wide range of industrial and technological sectors. Members are widely situated across Europe, with sales to EU customers exceeding € 1,000 billion, thereby sustaining around 6.6 million jobs in the region. European industry cannot flourish unless it can compete in a global economy. This capacity to compete cannot be determined solely by the efforts of individual companies. The prevailing economic and social policy framework is crucially important and must be flexible enough to adapt swiftly to changes in global conditions. ERT Member Companies' actions help to strengthen and support some of the key enabling conditions which trigger innovation and entrepreneurship elsewhere in the economy. Enabling conditions are part of the external business environment within which economic activity takes place, and are the result of actions undertaken by governments, public institutions and those in the private sector. ERT therefore advocates policies, at both national and European levels, which help create conditions necessary to improve European growth and jobs. [5]

The educational policy advocated by ERT in support of competitiveness is known as the MST Strategy (Mathematics, Science and Technology): "*ERT has identified increasing young people's interest in Mathematics, Science and Technology (MST) as essential for sustainable economic growth in Europe*." Its September 2009 report [6] states:

> MST plays a key role in growing adequate research and development (R&D) capacity, ensuring economic and productivity growth, and in other areas that are key to Europe's future competitive position. Europe needs more technology-driven highly skilled people to push back the frontiers of technology and drive innovation forward.
>
> The European Round Table of Industrialists (ERT) has continued to voice concern over the proportionate decline in the number of MST graduates and strongly believes that this issue must be tackled at a European Union level to ensure Europe's future prosperity. This report assesses business's role in the problem, brings together the input from stakeholders engaged in this issue within Europe, and puts forward a proposal for a European Coordinating Body. The European Coordinating Body will bring together the initiatives, build upon the knowledge and skills available to tackle the problem and help motivate young Europeans to study MST, thus helping equip young Europeans with the tools to influence and shape their future and help to ensure a more prosperous future for Europe. [11]

The European Union has set up a Framework programme FP7 for Research and Development which is currently under review in preparation for new FP8 program to start in 2014. The ERT's recommendation is that the Framework programme should:

> Strengthen the links between education, research and innovation (the knowledge triangle). More effective interaction between education and business is needed across Europe, in particular to enhance the necessary interest in mathematics, science and technology. An identification of best

> practice needs to be conducted in a systematic way and then spread within the EU. To this end, ERT supports the ongoing process of the creation of a European Coordinating Body.

A *Compendium of good practices in MST* report for the European Commission in 2010 [7] took as their exemplars: France, Latvia, the Netherlands, Norway, Portugal and Sweden. For example, Norway's strategy includes:

> New strategy to produce more scientists: More people have to decide to pursue educational programmes in mathematics, science and technology (MST) if Norwegian society is to develop in the desired direction. The Government is launching a national strategy to promote MST subjects [8].

Ironically it is the UK which is currently chairing the ERT's coordinating group on Mathematics, Science and Technology education. The current danger is that Education policy in England seems currently to be heading in a diametrically opposite direction to that of the rest of the Europe.

Within the UK, the Welsh Assembly Government has published its plans to develop its digital capacities in the document *Delivering a Digital Wales* [9], which outlines the contributions that public services, including schools will be expected to make.

Needless to say, the UK is not the only nation concerned about its economic future and the implications for the educational preparation of its future work force. There is a well-known presentation, originally designed by a US school teacher, called *Shift Happens* [10] which draws attention in a graphic fashion to the ways in which India and China put a particular focus on concentrating educational policy to make dramatic improvements in their output of skilled engineers, technologists and research scientists. Already we are seeing the results of their technological advances in news reports such of the *Indian tablet PC for education at $35* [11] and the *Chinese supercomputer* [12]. In 2006 the USA National Governors' Association established its Innovation America Task Force "*with the intent of joining efforts with the academic and business communities to strengthen the competitiveness of the United States in the global economy*". This has led to the development of the USA's STEM strategy, its 'Changing The Equation' initiative and the educational call to arms in President Obama's 2011 State of the Union address [13]. Michael Gove, in his June 2008 speech, whilst Shadow Secretary of State for Children, Schools, and Families, referred to exactly this point when he said:

> But looking at China and India's performance – indeed looking at the principal drivers of growth and innovation across the globe it seems increasingly clear to me that the answer to our problems, more often than not, is rocket science. It's rocket science – mathematics, engineering, physics and the other hard sciences which are driving innovation globally, and generating growth for the future. [14]

The Microsoft Research group's report *Towards 2020 Science* [15] clearly highlights the technical capacity, in the form of software and hardware, which is already available to support the solution of some of the world's most intractable problems, such as modelling the global warming of the planet and disease pandemics. However, the intellectual capacity to model such problems is a limiting factor, something for which the development of mathematics has a key role.

All computers rely on ingenious algorithms. For example, when searching for music on a hard drive amongst thousands of tracks the search engine does not simply go through the list one at a time in order, it uses much more efficient search techniques. These useful tools for solving particular problems are underpinned by many mathematical ideas. Although mostly invisible to the tool user, the mathematics of algorithms is central to the use of mobile electronic devices, computers and internet resources.

It is often argued that the number of mathematical *tool users* will outweigh many times the number of tool producers, with the implication that only a few experts need to really understand how to produce a mathematical tool. Such dichotomies are unhelpful as the workforce of the future will need to include a large number of mathematical tool modifiers. For the tool modifier, the central issues become the articulation of what the digital technology should do, designing a system to do it, and verifying that it will perform as specified.

Summary

4.1 A key issue facing the UK is how to inspire and develop the next generation of innovators, creators, scientists, technologists, engineers and mathematicians on which our future well-being and economy depends.

4.2 Policy makers at the European level state that increasing young people's interest in Mathematics, Science and Technology is essential for sustainable economic growth in Europe.

4.3 When comparing the UK to China and India Michael Gove said "It's rocket science – mathematics, engineering, physics and the other hard sciences which are driving innovation globally, and generating growth for the future" *(Gove, M. The answer is rocket science. 2008; Available from: http://conservativehome.blogs.com/torydiary/files/gove_on_science.pdf).*

4.4 There is a need to build on and improve the UK's capacity for technological innovation and creativity. Education at all levels has its part to play in engaging the interests and enthusiasm of young people so that they pursue education, training and career paths which both contribute to the nation's needs while themselves achieving satisfaction and reward.

5. How are digital technologies impacting on mathematical practices outside school and college?

Mathematicians have historically worked on the important practical issues of their era, which have always required both the development of the subject together with the tools to support this. Pascal's and Leibniz's mechanical calculating machines, Napier's logarithms, Babbage's difference engine, Newman's Colossus and Turing's Bombe for crypto-analysis at Bletchley Park are just a few examples of computational tools which have been fundamental to the evolution of digital technologies to support mathematical developments. In this respect mathematics is, and has always been, a dynamic problem solving activity for which humans have continued to develop and exploit new tools.

In the introductory Scenario we attempted to give a feel for a few of the ways in which DTs have been assimilated by the younger generation. People quickly forget, or never knew, that there was a world before photocopiers, fax machines, mobile phones, e-mails and the Internet. So it is hard to remember that the home ownership of computers was something deliberately provoked in the UK by a computer literacy strategy spearheaded by the BBC with its competitive tender to design and build a computer for family use at home. The BBC micro was released in 1981 and achieved overall sales of about 1.5 million both to schools and homes. This, together with its close successors such as the Sinclair ZX-Spectrum and the Commodore 64, introduced a generation of young people to computer programming – and inspired many of those to become involved with careers in which mathematics plays a major part. It was not until the third quarter of 2008 when sales of laptop computers overtook those of desktop PCs, but in the very short time since then we have seen a huge rise in the numbers of mobile computing devices such as e-books, 'pads' and smart-phones. The researchers for the 2007 Demos report *Their space – Education for a Digital Generation* [16] studied the daily use of DTs by a large group of children:

> The baseline finding from our research was that the use of digital technology has been completely normalised by this generation, and it is now fully integrated into their daily lives. The majority of young people simply use new media as tools to make their lives easier, strengthening their existing friendship networks rather than widening them. Almost all are now also involved in creative production, from uploading and editing photos to building and maintaining websites. However, we discovered a gap between a smaller group of digital pioneers engaged in groundbreaking activities and the majority of children who rarely strayed into this category. Meanwhile, contrary to society's assumptions about safety, this generation is also capable of self-regulation when kept well informed about levels of risk. Finally, many children we interviewed had their own hierarchy of digital activities when it came to assessing the potential for learning. In contrast to their teachers and parents they were very conscious that some activities were more worthwhile than others. [15]

The report draws out many important issues to be faced in thinking about future school education:

> The current generation of decision-makers – from politicians to teachers – see the world from a very different perspective from the generation of young people who do not remember life without the instant answers of the internet or the immediate communication of mobile phones. It is these decision-makers

> who shape the way that digital technologies are used in the system and who set them up to limit their use and role in everyday life. This is a short-term solution to a long-term change. In an economy driven by knowledge rather than manufacturing, employers are already valuing very different skills, such as creativity, communication, presentation skills and team-building. Schools are at the front line of this change and need to think about how they can prepare young people for the future workplace. But it is not just about schools – parents, young people and society in general have a blind spot in terms of recognising and valuing these 'softer' skills. [15]

The last forty years have seen an unprecedented change with the rise of the digital computer and associated communications technology. While *a computer* of the nineteenth century, literally a professional who performed arithmetic for a living, needed fluent and accurate mental arithmetic, the needs of the future workforce are rather different. For example, humans no longer add up lists of numbers, or search card records by hand. We rightly have machines to do this, but the techniques they use are becoming invisible to the people who use them.

Of course as we have already noted, DTs are the 'tools of the trade' of the modern scientists, technologists, engineers and mathematicians. In their book *Improving Mathematics at Work - The Need for Techno-Mathematical Literacies,* Hoyles, Noss and Kent consider the world of work from a mathematical viewpoint:

> What are the mathematical knowledge and skills that actually matter for the world of work today? Has technology reduced the necessary knowledge to the most basic arithmetic? Or has the era of globalised competition and customer-focus ushered in a new era where novel skills are required? If so, how can they be developed? This book argues that there has been a radical shift in the nature of mathematical skills required for work - a shift which has still not been fully recognised by either the formal education system or by employers and managers. People need mathematical knowledge and skills that are shaped in new ways by information technologies and situated in concrete work situations - what we term techno-mathematical literacies (TmL): for example, the need to be fluent in the language of mathematical inputs and outputs to technologies and to interpret and communicate them, rather than merely to be procedurally competent with calculations. [17]

So we must consider the issue of what experiences students at school, especially in the 11-19 range, should be given in the use of mathematical DTs and how best to develop the curriculum to enable this. We already have plenty of mathematical educational games where students gain rewards for effectively doing drill and practice in arithmetic in glossy surroundings. What we do not have is embedded practice in engaging students in using their skills with DTs to find out about, learn, apply and communicate aspects of mathematics. Nor have we yet managed to build in authentic use of mathematical DT tools such as for computer algebra, modelling, simulation, statistics, data-capture and 3D geometry.

What follows is a real example of what mathematical surprises the Internet can reveal. October 16th 2010 was Hamilton Day in Ireland – part of the annual Irish Maths Week. In preparing for a talk in Limerick, one of the working group needed to find out something about William Rowan Hamilton's mathematical contributions which might be relevant to young people of today. In searching for material about quaternions, he came across some interesting links. The NRICH website has some technical descriptions explaining what a quaternion is and how it can be used in transformation geometry [18]. The same article also a has a link to an article on the PLUS website called *Maths Goes To The Movies* by Joan Lasenby [19]. After introducing us to many of the mathematical techniques used for virtual reality in computer animation, including quaternions, the author concludes:

> If you've ever stayed to watch the entire credits roll you'll be aware that a huge variety of creative talent goes into making a successful movie: writers, directors, actors, costume designers, prop builders... the list goes on. But one name is often missed off that credit list — mathematics. Many of today's movies wouldn't be possible without the geometry of ray tracing or quaternions spinning objects in space. So the next time you settle into your cinema seat to enjoy a CG spectacle, raise your popcorn to mathematics, the hidden star of the show. [18]

The author then goes on to give a brief pen portrait of herself:

> Joan Lasenby read Mathematics at Trinity College Cambridge. This was followed by a PhD in the Physics department in the Radio Astronomy Group. After a brief spell in industry working for Marconi, she returned to academia and is now a University Lecturer in the Signal Processing Group of the Cambridge University Engineering Department and a Fellow and Director of Studies at Trinity College. Her research interests lie in the fields of computer vision, computer graphics, image processing, motion capture and geometric algebra. [18]

Summary

5.1 Mathematicians work on the important practical issues of their era, which have always required both the development of the subject together with the tools to support this. Pascal's and Leibniz's mechanical calculating machines, Napier's logarithms, Babbage's difference engine, Newman's Colossus and Turing's Bombe for crypto-analysis at Bletchley Park are just a few examples of computational tools which have been fundamental to the evolution of digital technologies to support mathematical developments. In this respect mathematics is, and has always been, a dynamic problem solving activity for which humans have continued to develop and exploit new tools.

5.2 New kinds of jobs are now open to people with mathematical qualifications, especially those with skills in the use of digital technologies. It is often argued that the number of mathematical tool users will outweigh many times the number of tool producers, with the implication that only a few experts need to really understand how to produce a mathematical tool. Such dichotomies are unhelpful as the workforce of the future will need to include a large number of mathematical tool modifiers. For the tool modifier, the central issue becomes the articulation of what the digital technology should do, designing a system to do it, and verifying that it will perform as specified.

5.3 The vast majority of young people are involved in creative production with digital technologies in their everyday lives, from uploading and editing photos to building and maintaining websites. They acquire many skills which will be relevant in their careers, but which are not drawn on during their time in school. They acquire new skills rapidly, and share their knowledge with their peers – but rarely in an educational context.

5.4 Whereas digital technologies are the ‘tools of the trade’ of the modern scientists, technologists, engineers and mathematicians, young people are not likely to use digital technologies for the creative production of science, technology and mathematics in their everyday lives.

Recommendation 1: For policy makers and teachers

School and college mathematics should acknowledge the significant use of digital technologies for expressive and analytic purposes both in mathematical practice outside the school and college and in the everyday lives of young people.

6. How are digital technologies impacting on mathematics education in the UK currently?

From the 1970s until the 1990s computer programming in some form became an element of school mathematics in the UK as exemplified by elements of the School Mathematics Project (1970s), Micro-electronics Education Programme Primary Logo project (1980s) and the Nuffield Advanced level mathematics curriculum (1990s). However with the introduction of 'information and communication technology' (ICT) as a curriculum subject the emphasis in schools shifted away from the strong link between mathematics and computing towards ICT being a 'subject' that incorporates aspects of communication, handling of information, and multimedia. While recognising that these general aspects of ICT are important, we urge that the specialised link between mathematics and computing in schools and colleges should not be lost, and should be strengthened.

Despite considerable investment in digital technologies at school level, successive Ofsted reports on the mathematics provision in secondary schools have concluded that the use of technology within mathematics is underused and, where it is used, its full potential is generally underexploited [20, 21]. Paragraphs 54-60 of the 2008 report *Understanding the score* [10] detailed the evidence revealed from school inspections and the report made the explicit recommendation that secondary schools should 'improve pupils' use of ICT as a tool for learning mathematics'. The evidence from the classroom suggests that the emphasis on the use of digital technology within school mathematics has been on teacher-led use, using mainly presentational software such as PowerPoint and interactive whiteboard software. Revision software and online content services are also used, with the focus being on the 'computer teaching mathematics' alongside practice exercises. Where digital mathematical tools such as graphing calculators, dynamic geometry, and spreadsheets are used, these are conceived primarily as presentational, visual and computational aids rather than as instruments to facilitate mathematical thinking and reasoning.

So the challenge for mathematics educators is to find ways which engage students with interesting, stimulating and challenging applications of mathematics which are relevant to their world. Of course there will always be a core of diligent students who will respond to the traditional stimulus of more abstract mathematics and for whom success will be its own reward. However unless we can develop mathematics education in a more stimulating way which takes cognisance of students' interests we are in danger of turning our subject into an increasingly 'dead language' and alienating groups of students whose mathematical potential will remain undeveloped.

The barriers to the more pupil-focussed use of digital technologies include:

- inadequate guidance concerning the use of technological tools within both statutory and non-statutory curriculum documentation;
- current assessment practices, which do not allow the use of digital technologies, particularly within high stakes assessments;
- a perception that digital technologies are an add-on to doing and learning mathematics.

The recent report from the National Centre for Excellence in Teaching Mathematics (NCETM) [22] concluded that mathematics teachers' concerns about the use of digital technologies related to:

- a lack of confidence with digital technologies;
- fears about resolving problems with the technology;
- fears about knowing less than their learners;
- access to digital technologies;
- inappropriate training;
- lack of time for preparation;
- a lack of awareness of how technology might support learning;
- not having technology use clearly embedded into schemes of work.

The report also recognised the importance of the role of the mathematics subject leader in supporting teacher development. More widely, the report acknowledged that professional development opportunities focussed on developing mathematics teachers' classroom uses of digital technologies needed to consider teachers' underlying attitudes and perceptions in addition to their technological skill development. It also concluded that the substantial research evidence reviewed suggested that collaborative and continued involvement with groups of peers, supported by appropriate expertise has proved successful in sustaining uses of digital technologies in the ways suggested within this report.

Within initial teacher education (ITE), higher education institutions and schools are required to validate trainees' evidence of personal ICT competence. Some, but not all, also embed a requirement that trainees should teach and evaluate a mathematics lesson in which their learners use digital technologies as part of an assessed course component. Research suggests that unless the school climate in which the trainees find themselves supports their further development, newly qualified teachers find it difficult to sustain more innovative practices [23].

Beyond mathematics, concern has been expressed about the universal teaching of generic ICT skills and the loss of more searching, specialised and relevant applications of digital technologies, which we have already noted as concerns of the Royal Academy of Engineering and Royal Society, within the Ofsted report *The importance of ICT* [24].

The mathematics curriculum and its assessment

At various times in the last fifty years or so the specification of subject content has changed to include, or to remove, specific topics such as binary arithmetic, Boolean logic, vectors, matrices, differential equations, flowcharting, complex numbers, linear programming and critical path analysis as well as applications such as electric circuits, particle kinematics, statistics and decision mathematics. The introduction of a national curriculum, together with regulatory authorities for public examinations and reductions in the number of examination boards, has led to a more stable but arguably less responsive, curriculum. In recent years the Royal Society has requested the Joint Mathematical Council to report on the teaching and learning of two specific aspects of mathematics: Algebra [25] and Geometry [26]. These reports were followed by a programme of work commissioned by the QCA [27, 28].

The JMC Algebra report, published in 1997, described two uses of technology to support teaching and learning algebra, namely spreadsheets and CAS:

> The second type of environment is the computer algebra system (CAS) (for example Derive and Mathematica) which is now available on hand-held calculators, which the majority of post-16 students are likely to own by the end of the century. There is relatively little research on how CAS can be used for the teaching and learning of school algebra and yet there is considerable 'hype' and pressure from commercial companies. Independent research studies are urgently needed to investigate some of the claims being made.

In the intervening years this resulted in a very small number of UK based pilot studies [29, 30], although internationally several countries (or autonomous regions) have taken up the positive opportunities provided by CAS with several countries requiring students to use CAS within mathematics examinations: Victoria, Australia; Bavaria, Germany; and Denmark. The place of CAS within the mathematics curriculum has been the topic of a recent seminar within the UK mathematics education community [31].

The JMC Geometry report recommended improvements in assessment approaches to allow for project work and also stated:

> ...that the geometrical content of the National Curriculum does provide a reasonable basis for the 11-16 curriculum, but needs strengthening in two main areas. These concern work in 3-dimensions, and in the educational application of Information and Communications Technology (ICT) [26].

Neither of these recommendations have been implemented and the inclusion of formally assessed extended project work and a greater emphasis in 3-D geometry are just a couple of examples of ways in which the mathematics curriculum might be developed to help support the skills needs of the future.

Currently at upper-secondary level, of the 90 distinct mathematical competences listed in the GCSE subject criteria for mathematics (for first teaching 2010), only 6 refer to the use of material tools [32]. Five of these references are to tools dating back many centuries: "use and interpret maps and scale drawings", "interpret scales on a range of measuring instruments", "draw… shapes using a ruler and protractor", "use straight edge and… compasses to do constructions", and "extract data from printed tables and lists". There is only one reference to a digital tool: "use calculators effectively and efficiently"; and, uniquely, use of this tool is barred for between 25 and 50 per cent of assessment. The one compulsory use of DTs, within GCSE data handling coursework, was abolished for students commencing study in 2008.

At lower secondary level, the current National Curriculum [33] does suggest that students should make use of "graphic calculators, dynamic geometry and spreadsheets" echoing recommendations in the earlier *Framework for Teaching Mathematics* [34]. Yet, while the Framework explicitly provides for students to learn to use manual tools as mathematical instruments (Figure 1**Error! Reference source not found.**), it generally overlooks this spect of using digital tools, with the solitary exception of the arithmetic calculator for which some provision is made.

For example, the knowledge and skill required to make use of a protractor to measure angles by hand is carefully specified. There is no equivalent guidance on how to measure an angle using dynamic geometry software [35].

Use a 180° or 360° protractor to measure and draw angles, including reflex angles, to the nearest degree. Recognise that an angle can be measured as a clockwise or anticlockwise rotation and that the direction chosen determines which will be the zero line and whether the inner or outer scale is to be used.

Figure 1 Extract from Framework for teaching mathematics: Year 7, 8, 9 [23]

In England and Wales, there are very few examples within the current formal assessment arrangements that involve the use of digital technologies by learners over and above the use of basic, scientific and graphing calculators. The related skills are limited to the order of operations within multi-step numerical problems and the use of built-in function keys such as square root or reciprocal.

A notable exception to this is the Mathematics in Education and Industry (MEI) AS/A–level syllabus, which includes a clear statement of its stance concerning the role of technology within mathematics.

> Mathematics has been transformed at this level by the impact of modern technology: the calculator, the spreadsheet and dedicated software. There are many places where this specification either requires or strongly encourages the use of such technology. The units DC and Numerical Computation have computer based examinations; options in FP2 and FP3 are based on graphical calculators, and the coursework in C3 and Numerical Methods is based on the use of suitable devices. [36]

However, although the MEI module ‘Further Pure 2’ offers an optional task to investigate curves on graphical calculators, the take-up of this module is very small. MEI's module ‘Numerical Methods’, which also has a coursework element requiring the use of spreadsheets, attracts about 1500 entries per year. The module ‘Numerical Computation’, within which the written examination is spreadsheet based, again has a very small take-up.

The close relationship between the formal assessment arrangements for mathematics and teachers’ resulting classroom practices may render any changes within the curriculum specification to include digital technologies as ineffective without a parallel consideration of how pupils’ use of mathematical digital technologies might be accredited. In post-16 education, there is scope for development within a number of existing specifications such as the Level 3 *Extended Project Qualification* [37], the AQA *Use of Maths* [38] and the International baccalaureate [39].

Teaching approaches

The way the mathematics curriculum is specified gives teachers a degree of autonomy with respect to the approaches they take to teaching and learning. This under-prescription has enabled some schools to innovate, whereas other schools have been more constrained by the current assessment arrangements. In the period between the publication of the Cockcroft Report *Mathematics Counts* [40] and the introduction of the National Curriculum, several major national projects provided vehicles for mathematics teachers to experiment with new teaching approaches as well as the use of digital technologies. These included the *Low Attainers in Mathematics Project* (LAMP) [41] and the *Raising Achievement in Mathematics Project* (RAMP) [42]. During this period the GCSE was introduced - with coursework as a compulsory element – and many examination groups offered professional development to mathematics teachers in supporting and assessing extended projects in mathematics, such as through the Southern Examination Group's *Teachers Evaluating and Assessing Mathematics* (TEAM). A significant curriculum development in this period was the development of *Mathematical Modelling for Schools* led by Cranfield, the Open University and the Shell Centre, as well as the *Project On Statistical Education* (POSE) from Sheffield – introducing new content, contexts and ways of working – often in group problem-solving. The mathematics teachers' professional associations, ATM and the MA, had active working groups producing resources, guidance and support for teachers introducing digital technologies into their teaching and learning.

The year 2000 saw both the UN International Year of Mathematics, with the UK's Maths Year 2000 website, and the extension of the National Numeracy Strategy's work from primary into secondary schools, with its *Framework for teaching mathematics: Year 7* [43]. It also saw the award of a contract by the DfEE to Research Machines, Oxford, to develop a Year 7 mathematics curriculum with a high ICT content called *MathsAlive!*. Soon afterwards the Strategy published a far more extensive Framework for Years 7, 8 and 9 [34] with an extensive section of examples including many innovative approaches using digital technologies including dynamic geometry software, graphical calculators and data-loggers. However, as there was no direct linkage with the kinds of questions on KS3 SATS and GCSE examination papers, there was little incentive for teachers to adopt these innovations. For several years now the main focus of professional development for English mathematics teachers has been on implementing the Secondary National Strategy, with a concentration on small groups of objectives. By contrast, the QCDA introduced a new National Curriculum in 2007 within a "big picture" of the curriculum with associated professional development in non-core subjects. Thus English secondary schools have been working on its implementation and attempting to join up most of the curriculum – but often with the notable exception of science, mathematics and ICT – the core of STEM.

Many schools do provide Enhancement and Enrichment (E&E) opportunities for students outside normal lesson time which can include opportunities for mathematical applications of digital technologies such as in STEM clubs and after-hours computing clubs for girls. Such activities provide non-threatening environments within which teachers can experiment, but to be effective there needs to some mechanism for the spread of the approach to all members of the mathematics department. The QCDA *Engaging Mathematics For All Learners 11-19* [44] provides case studies of many schools in which interesting opportunities for extended project work are offered, many of which involve the use of digital technologies, some of which are in a cross-curricular context such as STEM.

Developing the curriculum

In a visionary statement from 1985, the mathematics group convened by the Microcomputers in Education Programme (MEP) and chaired by Dr. Trevor Fletcher wrote:

> The relationship of mathematics to microcomputers is unique amongst the established school subjects in the following respects:
>
> - its historical relationship with developments in computing
> - the extent to which the subject matter is intimately related to the operation and working of the machine itself
> - the extent to which pupils can learn the subject by writing their own programmes.
>
> We consider that mathematical programming should be a staple part of the mathematics courses of the future. Just as a calculator has to be on hand if arithmetic is to be taught to best advantage, so a computer will be needed if algebra, geometry, statistics and other branches of mathematics are to be taught to the best advantage [45].

Other leading countries are engaging in curriculum development activities that recognise a major shift in the forms of mathematical and technical proficiency required by current learners. Such developments incorporate the use of digital technologies as integral to the curriculum and carefully consider the nature of the assessment approaches that promote the informed uses of digital technologies. For example, the European Schools have specified both the mathematics curriculum content and the associated techno-mathematical content within their 2010 curriculum, which has been approved by the relevant qualifications and curriculum authorities in each of the EU member states [46]. In its preamble, the document states,

> The syllabus preserves the foundations of mathematics teaching and leaves the core of the subject unchanged but at the same time it has as a new objective the systematic implementation of modern technological tools in the teaching. It also aims to create a common vehicle for teaching while allowing the teachers the freedom to introduce the fundamental concepts of the syllabus according to their own teaching methods.

The European Schools pedagogical office also specify the characteristics of the technological tools to be adopted in the implementation of its mathematics curriculum [47].

To summarise, what is currently missing in schools and colleges is student-led mathematical modelling, problem solving and computer programming which makes use of the powerful mathematical digital technologies that are widely used in society and the workplace.

Learning in science, technology and engineering in schools and colleges could be greatly enhanced if students were able to use digital technologies to perform mathematical processes, mirroring the types of applications used in STEM-based applications in the workplace. The benefit of using digital technologies relates both to the processing power afforded by the technologies and the opportunities to access real-world data, which is engaging for students.

There is already a wide range of existing digital technologies which could readily be applied by many more schools and colleges such as:

- Dynamic graphing tools
- Dynamic geometry tools
- Algorithmic programming languages
- Spreadsheets
- Data handling software and Dynamic statistical tools
- Computer algebra systems
- Data loggers, such as motion detectors and GPS
- Simulation software

For example, the use of a dynamic geometry tool by students in which they learn about the angle properties of circles is exemplified within the Teacher's TV programme *Hard to Teach – Secondary maths using ICT* [48].

Increasingly the specification of modern laptop and e-book portable computers is such that virtually all of the major mathematical technologies now run on them. Similarly there have been significant improvements in broadband Internet connections and collaborative tools. The expansion of pupils' access to technology is less of an issue than the leadership and management of the resources at school and college level. If digital technologies are to be embedded in the mathematics curriculum then the knowledge and skills required by both teachers and learners to use them should be explicitly specified.

The current review of the National Curriculum provides an ideal opportunity for industry, subject associations and schools to collaborate to develop a mathematics curriculum that is fit for purpose and embraces the use of digital technologies.

With respect to the content of the new curriculum, it should include:

- a component of computer programming, interpreted in the widest sense of creating and communicating a set of instructions to a computer for a clear purpose.
- the specification of the knowledge and skills required to use digital technologies within modelling and problem solving activities across a range subject areas.
- student-led mathematical modelling and problem solving, which make use of the powerful mathematical digital technologies that are widely used in society and the workplace.

Change to the mathematics curriculum and its assessment, although necessary, will not be sufficient to develop the classroom practices of teachers, many of whom have limited knowledge and experience of the type of digital technological tools being described in this report. As the development of a technologically enriched student learning experience occurs at the level of the classroom, such change has to be supported by both school leaders and accompanied by sustained professional development opportunities for teachers. The process of adopting digital technologies requires teachers to take risks in their teaching, supported by appropriate technical, didactical and mathematical expertise. With increased autonomy, schools will need to take more responsibility themselves for professional development and support for teachers in this process. With the demise of some of the government funded agencies, the expertise and commitment of the teachers' subject professional associations will be an invaluable resource.

The development of electronic professional development resources for teachers, possibly offered as 'e-CPD' is an approach that has been explored (but not exploited), previously through the *Enhancing subject teaching using ICT* initiative, which evolved into the *Practical Support Pack* [49] both developed by the mathematics subject associations and funded by the DfEE/DCSF. More recently, companies such as Intel and Cisco are developing DT systems to support teacher development and school collaboration. The NCETM report *Mathematics and digital technologies: New Beginnings* [22] also acknowledged the need for improved access to professional development for teachers. Acting on its own recommendation, the NCETM commissioned two mathematics subject associations to produce training and materials for those involved in initial teacher education and authority-wide professional development. In addition, tools such as *Elluminate*, *Skype*, *Jing* and *Windows Live* now facilitate a variety of ways for teacher/teacher, teacher/student and student/student interactions at a distance. The mathematics subject associations are well placed to develop this means of applying DTs to support the professional development of mathematics teachers in embedding digital technologies.

Summary

6.1 Despite considerable investment in digital technologies in schools, successive Ofsted reports have concluded that the use of technology within mathematics is underused and, where it is used, its potential is generally underexploited. The emphasis on the use of digital technology within school mathematics has been on teacher-led use, using mainly presentational software such as PowerPoint and interactive whiteboard software.

6.2 Unless we can develop mathematics education in a more stimulating way, which takes into account the modern world and students' interests we are in danger of turning mathematics into an increasingly 'dead language' and alienating groups of students whose mathematical potential will remain undeveloped.

6.3 Rote learning of the current mathematics curriculum will not be sufficient to produce the problem solvers, independent thinkers and designers that the country needs.

6.4 The barriers to a more creative student-focussed use of digital technologies include:

- a perception that digital technologies are an add-on to doing and learning mathematics;
- current assessment practices, which do not allow the use of digital technologies, particularly within high stakes assessments;
- inadequate guidance concerning the use of technological tools within both statutory and non-statutory curriculum documentation.

6.5 Learning in science, technology and engineering in schools and colleges could be greatly enhanced if students were able to use digital technologies to perform mathematical processes, mirroring the types of applications used in STEM-based applications in the workplace. The benefit of using digital technologies relates both to the processing power afforded by the technologies and the opportunities to access real-world data, which is engaging for students.

6.6 There is already a wide range of existing 'mathematical' digital technologies which could readily be used by schools and colleges such as:

- Dynamic graphing tools.
- Dynamic geometry tools.
- Algorithmic programming languages.
- Spreadsheets.
- Data handling software and dynamic statistical tools.
- Computer algebra systems.
- Data loggers, such as motion detectors and GPS.
- Simulation software.

6.7 With respect to the content of the new mathematics curriculum, it should include:

- the specification of the knowledge and skills required to use digital technologies within mathematical modelling and problem solving activities across a range of subject areas;
- student-led mathematical modelling and problem solving, which make use of the powerful mathematical digital technologies that are widely used in society and the workplace;
- a component of computer programming, interpreted in the widest sense of creating and communicating a set of instructions to a computer for a clear purpose.

6.8 Change to the mathematics curriculum and its assessment, although necessary, will not be sufficient to develop the classroom practices of teachers, many of whom have limited professional knowledge and experience of the type of digital technological tools being described in this report.

6.9 There will be a continuing need to update the skills of the teaching workforce, something which the subject associations and teacher education institutions working alongside industry and schools are well-placed to achieve.

Recommendation 2: For policy makers

Curriculum and assessment in school mathematics should explicitly require that all young people become proficient in using digital technologies for mathematical purposes.

Recommendation 3: For policy makers

High-stakes assessment needs to change in order to encourage the creative use of digital technologies in mathematics classes in schools and colleges.

Recommendation 4: For policy makers and school leaders

As the development of a technologically enriched student learning experience occurs at the level of the classroom, such change has to be supported by school leaders and accompanied by sustained professional development opportunities for teachers.

Recommendation 5: For policy makers

The UK Departments for Education and for Business, Innovation and Skills should establish a Task Force to take the lead in bringing together various parties with appropriate expertise to take forward the recommendations of this report and advise the Departments on required policy initiatives.

7. References

1. Royal Academy of Engineering, *ICT for the UK's future: the implications for the changing nature of Information and Communications Teachnology*. 2009, London: Royal Academy of Engineering.
2. Dyson, J. *Ingenious Britain: Making the UK the leading high tech exporter in Europe*. March 2010.
3. Science and Learning Expert Group, *Science and Mathematics Secondary Education for the 21st Century*. 2010, Department for Business, Innovation and Skills: London.
4. Livingstone, I. and A. Hope, *Next Gen. Transforming the UK into the world's leading talent hub for the video games and visual effects industries*. 2010, Nesta: London.
5. European Round Table of Industrialists. 2011; Available from: http://www.ert.be/home.aspx.
6. European Round Table of Industrialists, *Mathematics, Science and Technology Report: Case for a European Coordinating Body.* 2009, European Round Table of Industrialists: Brussels.
7. European Commission, *Compendium of good practices in MST*. 2010, European Commission: Brussels.
8. The Norwegian Collobration Centre. *Innovation: Cultivating great ideas*. Available from: http://www.norway2uk.com/en/innovation/.
9. Welsh Assembly Government, *Delivering a digital Wales. The Welsh Assembly Government's Outline Framework for Action*. 2010, Welsh Assembly Government: Cardiff.
10. Fisch, K. *Did you know: Shift happens*. 2008; Available from: http://www.public.iastate.edu/~mcleod/didyouknow/.
11. British Broadcasting Corporation, *India unveils prototype for $35 touch-screen computer*, in *BBC News*. 2010, BBC.
12. British Broadcasting Corporation, *China claims supercomputer crown*, in *BBC News*. 2010, BBC.
13. Obama, B. *The State of the Union Address 2011: Winning the Future*. 2011; Available from: http://www.whitehouse.gov/the-press-office/2011/01/25/remarks-president-state-union-address.
14. Gove, M. *The answer is rocket science*. 2008; Available from: http://conservativehome.blogs.com/torydiary/files/gove_on_science.pdf
15. The 2020 Science Group, *Towards 2020 Science*. 2006, Microsoft Research: Cambridge, UK.
16. Green, H. and C. Hannon, *Their space – Education for a Digital Generation*. 2007, Demos: London.
17. Hoyles, C., et al., *Improving mathematics at work - The need for techno-mathematical literacies*. 2010, Abingdon: Routledge.
18. nrich. *Quaternions and Reflections*. Available from: http://nrich.maths.org/5628.
19. Lasenby, J. *Maths goes to the movies*. 2007; Available from: http://plus.maths.org/content/os/issue42/features/lasenby/index.
20. Office for Standards in Education, *Evaluating mathematics provision for 14–19-year-olds*. 2006, Her Majesty's Inspectorate: London.
21. Office for Standards in Education, *Mathematics: Understanding the Score*. 2008, Department for Children, Schools and Families: London.
22. National Centre for Excellence in the Teaching of Mathematics, *Mathematics and digital technologies: New Beginnings*. 2010.
23. Hammond, M., et al., *What happens as student teachers who made very good use of ICT during pre-service training enter their first year of teaching?* Teacher Development, 2009. **13**(2): p. pp. 93-106.
24. Office for Standards in Education, *The importance of ICT*. 2009, London: Office for Standards in Education.

25. The Royal Society, ed. *Teaching and learning algebra pre-19*. 1997, The Royal Society: London.
26. The Royal Society, *Teaching and learning geometry 11-19*. 2001, The Royal Society: London.
27. Qualifications and Curriculum Authority, *Developing reasoning through algebra and geometry*. 2004, London: Department for Education and Skills.
28. Brown, M., K. Jones, and R. Taylor, *Developing geometrical reasoning in the secondary school: outcomes of trialling teaching activities in classrooms. A report from the Southampton/Hampshire group to the Qualifications and Curriculum Authority*. 2003, University of Southampton: Southampton.
29. Lumb, S., J. Monaghan, and S. Mulligan, *Issues arising when teachers make extensive use of computer algebra in their mathematics lessons.* International Journal of Computer Algebra in Mathematics Education, 2000. **7**(4): p. 223-240.
30. Monaghan, J., *Some issues surrounding the use of algebraic calculators in traditional examinations.* International Journal of Mathematical Education in Science & Technology, 2000. **31**(3): p. 381-392.
31. MEI, *Computer algebra systemes and the mathematics curriculum: Report of the invitation seminar (May 2008)*. 2008, Texas Instruments: Northampton, UK.
32. Office of Qualifications and Examinations Regulation, *GCSE subject criteria for mathematics*. 2009, Ofqual: London.
33. Department for Children Schools and Families, *National Curriculum for mathematics*. 2007, Qualifications and Curriculum Agency: London, UK.
34. Department for Education and Employment, *The National Numeracy Strategy. Framework for teaching mathematics: Year 7, 8 and 9*. 2001, HMSO: London.
35. Ruthven, K., *The Interpretative Flexibility, Instrumental Evolution, and Institutional Adoption of Mathematical Software in Educational Practice: The Examples of Computer Algebra and Dynamic Geometry.* Journal of Educational Computing Research, 2008. **39**(4): p. pp. 379-394.
36. Oxford Cambridge and RSA Examinations, *MEI Structured Mathematics Specification*. 2003, MEI/OCR: Cambridge.
37. Office of Qualifications and Examinations Regulation. *Foundation, Higher and Extended Project Criteria*. 2010; Available from: http://www.ofqual.gov.uk/qualification-and-assessment-framework/89-articles/527-foundation-higher-and-extended-project-criteria.
38. Office of Qualifications and Examinations Regulation. *AQA Level 3 Advanced Subsidiary GCE in Use of Mathematics*. Available from: http://register.ofqual.gov.uk/Qualification/Details/100_1330_1.
39. International Baccalaureate Organisation. *Diploma Programme curriculum - Group 5: Mathematics and computer science*. Available from: http://www.ibo.org/diploma/curriculum/group5/.
40. Cockcroft, W., *Mathematics counts*. 1982, Department of Education and Skills: London.
41. Ahmed, A., *Better mathematics*. 1987, Her Majesty's Stationery Office: London.
42. Ahmed, A. and H. Williams, *Raising Achievement in Mathematics Project: A curriculum development research project*. 1981, The Mathematics Centre, Chichester Institute of Higher Education: Bognor Regis.
43. Department for Education and Employment, *The National Numeracy Strategy. Framework for teaching mathematics: Year 7*. 2000, HMSO: London.
44. Qualifications and Curriculum Authority, *Engaging Mathematics for all Learners*. 2009, Qualifications and Curriculum Authority: London.
45. Fletcher, T.J., *Pendly Manor Report: Mathematics and microcomputers.* Mathematics in School, 1985. **14**(2).
46. European Schools. *Mathematics syllabus Year 4 to Year 7*. 2010; Available from: http://www.eursc.eu/index.php?id=149.

47. European Schools. *Mathematics, years 4 to 7 - Characteristics of the technological tool to be implemented*. 2010; Available from: http://www.eursc.eu/index.php?id=149.
48. Teachers TV. *Hard to Teach - Secondary Maths Using ICT*. 2008; Available from: http://www.teachersmedia.co.uk/videos/secondary-maths-using-ict.
49. Department for Education and Skills (2003) *Practical Support Pack: Secondary Mathematics*.

8. Membership of the working group

Professor Janet Ainley, School of Education, University of Leicester

Tom Button, Further Mathematics Support Programme

Dr Alison Clark-Wilson, The Mathematics Centre, University of Chichester

Dr Steve Hewson, Department of Applied Mathematics and Theoretical Physics, University of Cambridge

Dr Sue Johnston-Wilder, Institute of Education, University of Warwick

Dr David Martin, Executive Secretary, Joint Mathematical Council of the UK

Dave Miller, Keele University

Professor Adrian Oldknow, University of Chichester

Professor Kenneth Ruthven, Faculty of Education, University of Cambridge

Dr Chris Sangwin, School of Mathematics, University of Birmingham

Neil Sheldon, Manchester Grammar School

Professor Rosamund Sutherland (Chair), Graduate School of Education, University of Bristol

Members of the working group:
Professor Janet Ainley, School of Education, University of Leicester
Tom Button, Further Mathematics Support Programme
Dr Alison Clark-Wilson, The Mathematics Centre, University of Chichester
Dr Steve Hewson, Department of Applied Mathematics and Theoretical Physics, University of Cambridge
Dr Sue Johnston-Wilder, Institute of Education, University of Warwick
Dr David Martin, Executive Secretary, Joint Mathematical Council of the UK
Dave Miller, Keele University
Professor Adrian Oldknow, University of Chichester
Gill Potter, Independent Consultant
Professor Kenneth Ruthven, Faculty of Education, University of Cambridge
Dr Chris Sangwin, School of Mathematics, University of Birmingham
Neil Sheldon, Manchester Grammar School
Professor Rosamund Sutherland, Graduate School of Education, University of Bristol

Acknowledgements:
We thank the following schools for the photographs included in this report:
Davison CE High School for Girls, Worthing, West Sussex; Fairfield High School, Bristol; Bacon's College, Southwark.
We also thank Conrad Wolfram and Douglas Butler for providing images.
Design: cw-design.co.uk

www.jmcuk.org.uk